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State estimation of switched systems

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1 Project Aim

In this project, the state estimation of switched systems with noise disturbances is considered. Switched systems may change their dynamics as well as their input and output configuration according to an operation mode σ. The estimation problem is to reconstruct the current state $\boldsymbol{x}(k)$ at time k from the given input and output sequences and the measured operation mode. This problem can be solved by the use of a switching estimation scheme where the current filter is switched synchronously to the system. If at time k a switch of the system mode occurs, i.e. $\sigma(k) \rightarrow \bar{\sigma}(k)$, a new filter is initialised with the current estimate $\hat{\boldsymbol{x}}_k$ and the covariance $\boldsymbol{P}_k$ of the estimation error $\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k$. This behaviour is depicted in figure 1 where the considered switched system consists of two subsystems which are differing in the number of available measurement data. The analyse of stability and convergence of this switching estimation algorithm is the main purpose of this project.

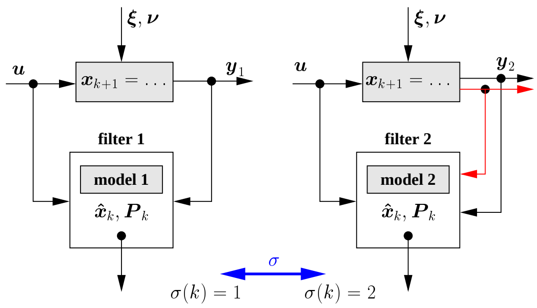

Figure 1: Switching state estimation due to a changing measurement structure

Switched systems are occurring in many real world applications. A typical example is the estimation of the traffic state on a freeway section by means of Floating-Car-Data (FCD) [1]. Induction loops count continuously the number and the velocity of the passing vehicles. Additional information is provided by single vehicles which transmit from time to time their actual position and velocity to the traffic control centre as it is depicted in figure 2. The control of a simulated countercurrent chromatographic separation process may serve as another example [5].

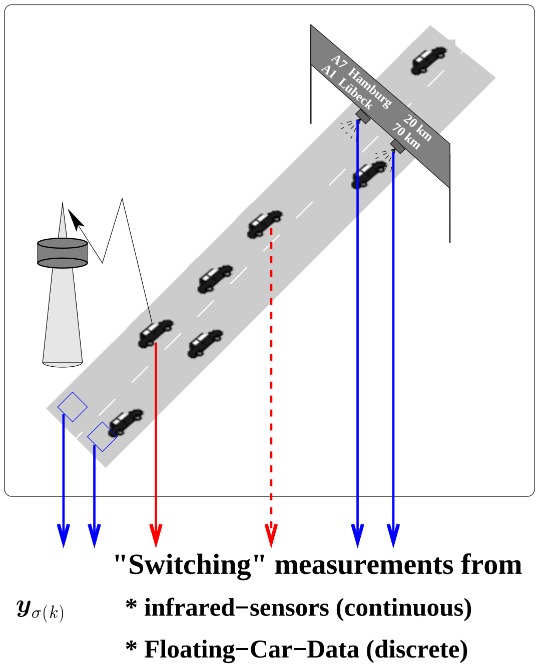

Figure 2: Traffic state estimation with FCD

2 Switched affine model

Switched affine systems are represented by the discrete-time model

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \bar{\boldsymbol{f}}_{\sigma(k)} + \boldsymbol{A}_{\sigma(k)}\boldsymbol{x}_k + \boldsymbol{B}_{\sigma(k)}\boldsymbol{u}_{\sigma(k)} + \boldsymbol{\Gamma}_{\sigma(k)}\boldsymbol{\xi}_{\sigma(k)} \\ \boldsymbol{y}_{\sigma(k)} &= \bar{\boldsymbol{g}}_{\sigma(k)} + \boldsymbol{C}_{\sigma(k)}\boldsymbol{x}_k + \boldsymbol{\nu}_{\sigma(k)} \qquad (1) \\ \sigma(k) &\in I = \{1, \dots, q\}, \end{aligned}$$

where the switching signal $\sigma(k)$ defines the *system* or *operation mode* at the time instant $k \cdot T$, which determines the current system dynamics and the structures of the measurement and the input vectors. The signal $\sigma(k)$ may be described by a finite discrete-event automaton. The number q of different system modes

σ is supposed to be finite. Hence, $\sigma \in I = \{1, \ldots, q\}$ holds. Furthermore, the order of the switched system (1) should be equal to n for all modes $\sigma \in I$. Each system matrix $\boldsymbol{A}_\sigma$ is assumed to be invertible. The numbers of inputs and of outputs are allowed to depend on the mode σ, which also changes the dimensions of $\boldsymbol{B}_{\sigma(k)}$ and $\boldsymbol{C}_{\sigma(k)}$. It is assumed that the unbiased, white noise Gaussian stochastic processes $\boldsymbol{\xi}_{\sigma(k)}$ and $\boldsymbol{\nu}_{\sigma(k)}$ are uncorrelated to each other and to the unknown initial state $\boldsymbol{x}_0$. Their statistical properties are described by the positive definite covariances $\boldsymbol{Q}_{\sigma(k)}$ and $\boldsymbol{R}_{\sigma(k)}$ which are supposed to be known for each operation mode $\sigma \in I$. Finally, it is assumed that the state $\boldsymbol{x}_k$ does not jump during the change of the operation mode.

3 Problems and solutions

It is a known fact that the stability of a switched system cannot be deduced from the stability of the involved subsystems. Vice versa, a switched system may be stabilised by an adequate sequence of operation modes σ although all subsystems are unstable [3]. The problems occurring in the state reconstruction of switched systems are illustrated in [4], where an example of a switched observer designed by pole placement is presented. Although the observers for all operation modes are stable, the switching observation scheme leads to an infinitely growing estimation error. However, it is also demonstrated that the switching observer can be stabilised if all observers satisfy additional requirements which guarantee the monotonic decrease of the estimation error for each operation mode.
The state estimation $\hat{\boldsymbol{x}}_k$ of a switched system is obtained by the following estimation scheme:

$$\begin{cases} \boldsymbol{x}^*_{k+1} &= \bar{\boldsymbol{f}}_{\sigma(k)} + \boldsymbol{A}_{\sigma(k)}\hat{\boldsymbol{x}}_k + \boldsymbol{B}_{\sigma(k)}\boldsymbol{u}_{\sigma(k)} \\ \hat{\boldsymbol{x}}_{k+1} &= \boldsymbol{x}^*_{k+1} + \boldsymbol{L}_{\sigma(k+1)}(\boldsymbol{y}_{\sigma(k+1)} - \boldsymbol{C}_{\sigma(k+1)}\boldsymbol{x}^*_{k+1}) \end{cases}$$

For each subsystem $\sigma \in I$ a Kalman filter algorithm may be used to calculate the filtering gains $\boldsymbol{L}_{\sigma(k)}$. In [2] it is proved that the use of an optimal filtering algorithm guarantees the boundedness and optimality of the resulting estimation error provided that each subsystem $\sigma \in I$ is observable and controllable. A switching Lyapunov function shows then the stability of the estimation error. Nevertheless, the optimality of the filtering algorithm requires the solution of a matrix Riccati equation

$$\begin{aligned} \boldsymbol{P}_{k+1} = \boldsymbol{A}_{\sigma(k)}\boldsymbol{P}_{\sigma(k)}\boldsymbol{A}^T_{\sigma(k)} - \boldsymbol{A}_{\sigma(k)}\boldsymbol{P}_k\boldsymbol{C}^T_{\sigma(k)} \cdot \\ (\boldsymbol{C}_{\sigma(k)}\boldsymbol{P}_k\boldsymbol{C}^T_{\sigma(k)} + \boldsymbol{R}_{\sigma(k)})^{-1}\boldsymbol{C}_{\sigma(k)}\boldsymbol{P}_k\boldsymbol{A}^T_{\sigma(k)} \\ +\boldsymbol{\Gamma}^T_{\sigma(k)}\boldsymbol{Q}_{\sigma(k)}\boldsymbol{\Gamma}^T_{\sigma(k)}, \end{aligned}$$

at each time step k for the computation of the time-variant filtering gains $\boldsymbol{L}_{\sigma(k)}$. Therefore, it is interesting to search for time-invariant filtering gains $\bar{\boldsymbol{L}}_\sigma$ for each operation mode. However, in [2] an example is given, where the use of the limiting Kalman filter solutions of each operation mode leads to an infinitely growing estimation error.
In this project, the switching state estimation is applied to the problem of traffic state estimation on a freeway section. On the basis of real measurement data from induction loops it is tested if additional FCD information may improve the results of the state estimation. In figure 3 some experimental results are depicted. It could have been shown that a FCD information rate of 2 *min.* could compensate the failure of an induction loop.

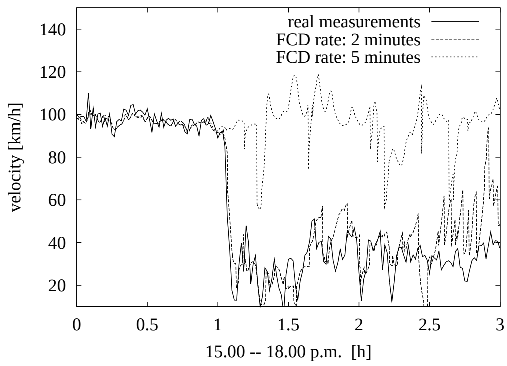

Figure 3: Real measured velocity and filter estimation depending on different FCD rates

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