RUHR-UNIVERSITÄT BOCHUM
Institute of Automation and Computer Control
Prof. Dr.-Ing. Jan Lunze

Universitätsstr. 150 Phone +49-(0)234 32-28071
D-44805 Bochum Fax +49-(0)234 32-14101

Feedback Controller Design for Nondeterministic I/O Automata

Melanie Schuh
melanieschuh@atp.rub.de

1 Project aim

In this project the control of discrete event systems modeled by nondeterministic Input/Output (I/O) automata is considered. The control aim is to steer the system from its initial state into a desired final state $z_{\rm F}$ by applying an appropriate input sequence. In order to allow the controller $\mathcal{C}$ to react to the motions of the plant $\mathcal{P}$, both are connected in a feedback structure (Fig. 1). It is aimed to define the controller such that the closed-loop system $\overline{\mathcal{P}}$ can be described by an I/O automaton to enable an analytical evaluation of the behavior of the control loop. Within this framework, the following questions have to be answered:

Controllability: Can the plant $\mathcal{P}$ be steered into the desired final state $z_{\rm F}$?

Controller design: How to design a controller $\mathcal{C}$ which steers the plant $\mathcal{P}$ into the desired final state $z_{\rm F}$?

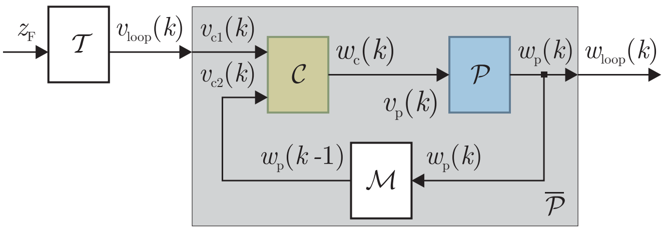

Figure 1: Feedback control of nondeterministic automata

2 Control framework

Plant $\mathcal{P}$. The plant $\mathcal{P}$ to be controlled is modeled by a nondeterministic I/O automaton

$$\mathcal{N}_{\rm p} = (\mathcal{Z}_{\rm p}, \mathcal{V}_{\rm p}, \mathcal{W}_{\rm p}, L_{\rm p}, z_{\rm p0})$$

with the state set $\mathcal{Z}_{\rm p}$, the input set $\mathcal{V}_{\rm p}$, the output set $\mathcal{W}_{\rm p}$, the behavioral relation $L_{\rm p}$ and the initial state $z_{\rm p0}$. $L_{\rm p}(z'_{\rm p}, w_{\rm p}, z_{\rm p}, v_{\rm p})$ has the value one, if the system can go from state $z \in \mathcal{Z}_{\rm p}$ with input $v_{\rm p} \in \mathcal{V}_{\rm p}$ to state $z'_{\rm p} \in \mathcal{Z}_{\rm p}$ and thereby produce the measurable output $w_{\rm p} \in \mathcal{W}_{\rm p}$. The initial state $z_{\rm p0}$ of the plant is assumed to be known unambiguously.

Controller $\mathcal{C}$. The controller $\mathcal{C}$ is modeled as a deterministic I/O automaton

$$\mathcal{A}_{\rm c} = (\mathcal{Z}_{\rm c}, \mathcal{V}_{\rm c}, \mathcal{W}_{\rm c}, G_{\rm c}, H_{\rm c}, z_{\rm c0}), \qquad (1)$$

where $G_{\rm c} : \mathcal{Z}_{\rm c} \times \mathcal{V}_{\rm c} \to \mathcal{Z}_{\rm c}$ is the state transition function of the controller and $H_{\rm c} : \mathcal{Z}_{\rm c} \times \mathcal{V}_{\rm c} \to \mathcal{W}_{\rm c}$ is the output function of the controller.

As can be seen from Fig. 1, the input $\boldsymbol{v}_{\rm c}$ to the controller is a vector consisting of two elements, $v_{\rm c1}$ and $v_{\rm c2}$. The first input $v_{\rm c1}$ is generated by the trajectory planning unit $\mathcal{T}$, while the second input $v_{\rm c2}$ equals the output of the memory block $\mathcal{M}$. Within the considered setting, the output $w_{\rm c}$ of the feedback controller automaton becomes the input $v_{\rm p}$ of the plant.

Memory block $\mathcal{M}$. To reflect the time delay existent in the physical plant, a memory block $\mathcal{M}$ which delays the output of the plant by one time step is included into the feedback path between the plant and the controller. The memory block $\mathcal{M}$ can be modeled as a deterministic Moore automaton

$$M = (\mathcal{Z}_{\rm M}, \mathcal{V}_{\rm M}, \mathcal{W}_{\rm M}, G_{\rm M}, H_{\rm M}, z_{\rm M0}).$$

Trajectory planning unit $\mathcal{T}$. The trajectory planning unit $\mathcal{T}$ generates a desired trajectory $Z_{\rm s}(0 \ldots k_{\rm e}+1)(0 \ldots k_{\rm e}+1) = (z_{\rm p0}, \ldots, z_{\rm F})$ for the plant, which is found by means of the search for a path from the initial state $z_{\rm p0}$ of the plant to the desired final state $z_{\rm F}$ in the model $\mathcal{N}_{\rm p}$ of the plant. The elements within this trajectory then become the inputs $v_{\rm c1}(k) = z_{\rm s}(k+1)$ to the controller $\mathcal{C}$.

Model of the closed-loop system $\overline{\mathcal{P}}$. As all elements within the gray box in Fig. 1 are modeled as I/O automata, the resulting closed-loop system can be described by a nondeterministic I/O automaton $\mathcal{N}_{\rm loop}$ with input $v_{\rm loop}(k) = v_{\rm c1}(k)$ and output $w_{\rm loop}(k) = w_{\rm p}(k)$. Therefore, its behavior in dependence on the planned trajectory $Z_{\rm s}(0 \ldots k_{\rm e}+1)(0 \ldots k_{\rm e}+1)$ can be easily analyzed and properties such as the controllability of the plant $\mathcal{P}$ can be proven.

Interaction of the components. The aim of the controller $\mathcal{C}$ to be designed is to steer the plant $\mathcal{P}$ along the given trajectory $Z_{\rm s}(0 \ldots k_{\rm e}+1)(0 \ldots k_{\rm e})$. However, due to its nondeterminism, the plant may deviate from the desired trajectory. It is assumed that the trajectory planning unit selects the trajectory such that at every time step the controller has the chance to return the plant to the trajectory, which might or might not succeed. Furthermore, the trajectory has to be such that in the last step, when the final state $z_{\rm F}$ is to be reached, no deviation is obtained.

3 Controller design

The controller is defined such that it aims at steering the plant along the desired trajectory $Z_s(0 \dots k_e + 1)(0 \dots k_e)$ generated by the trajectory planning unit $\mathcal{T}$ at every time step. To deal with the nondeterministic behavior of the plant, the controller contains an observer determining the set $\mathcal{Z}_{\text{poss}}$ of possible current states of the plant and makes its control decisions based on this set (Fig. 2).

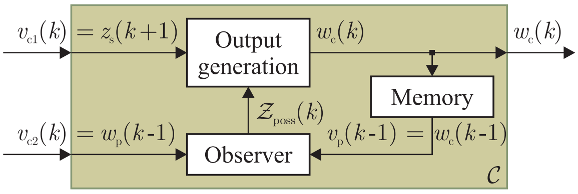

Figure 2: Controller mechanism

Formally, the controller is defined as a deterministic I/O automaton $\mathcal{A}_c$ as in (1), whose state stores the set $\mathcal{Z}_{\text{poss}}$ as well as the last input $v_p = w_c$ that has been applied to the plant. Its output function $H_c(z_c, [v_{c1}, v_{c2}]^T)$ generates an output w_c that is able to steer the plant from any state in the set $\mathcal{Z}_{\text{poss}}$ into the next state $z_s = v_{c1}$ specified by the trajectory planning unit $\mathcal{T}$. The transition function G_c then updates the controller state z_c such that it again reflects the current knowledge about the plant state after the application of the input $v_p = w_c$ to it.

It has been shown in [1] that a controller $\mathcal{C}$ designed with this method always steers a controllable plant $\mathcal{P}$ into the desired final state z_F, if it receives the correct trajectory $Z_s(0 \dots k_e)$ from the trajectory planning unit $\mathcal{T}$.

4 Example: Control of a workcell

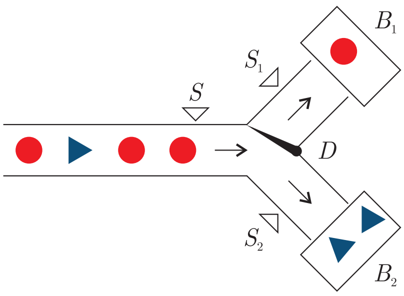

Figure 3: Plant sorting workpieces

System description. Consider a very simple plant that can sort workpieces into two different boxes, B_1 and B_2 (Fig. 3). There are two kinds of workpieces, circular workpieces and triangular workpieces, which shall be moved into box B_1 and box B_2, respectively. The classification and sorting of a single workpiece is represented by the automaton graph of $\mathcal{N}_p$ in Fig. 4. In the plant automaton, the nondeterminism can be seen in state $z_p = 1$ when the new workpiece can either be a circular or a triangular one. State $z_p = 4$ reflects the situation, where either a circular workpiece has been transported to box B_1 or a triangular workpiece has been moved to box B_2.

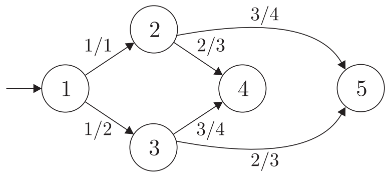

Figure 4: Nondeterministic plant automaton $\mathcal{N}_p$

Controller design. The graph of the corresponding controller automaton is shown in Fig. 5. It can be seen that the second state of the controller contains two plant states, namely $\{2, 3\}$, which means that the state of the plant is not known exactly at this time.

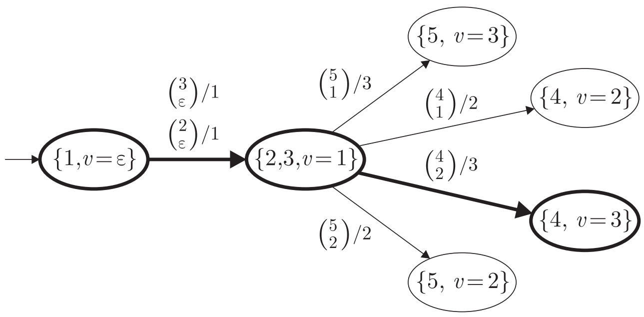

Figure 5: Automaton $\mathcal{A}_c$ of the controller $\mathcal{C}$

Behavior of the closed-loop system. The control aim is to steer the plant from its initial state $z_{p0} = 1$ into the desired final state $z_F = 4$ and to thereby sort every workpiece into the correct box. Therefore, for example, the trajectory $Z_s(0 \dots 2) = (1, 3, 4)$ could be specified by the trajectory planning unit $\mathcal{T}$. If a circular workpiece enters the plant, the plant follows the state sequence $Z_p(0 \dots 2) = (1, 2, 4)$. It can be seen that the plant temporarily deviates from its desired behavior. However, as only the achievement of the final state $z_F = 4$ is required, this deviation can be regarded as a small disturbance that can be rejected by the controller $\mathcal{C}$.

References

[1] M. Schmidt and J. Lunze. Feedback control of nondeterministic Input/Output automata. In *53rd IEEE Conference on Decision and Control*, 2014. submitted.