Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 28071
Fax +49 - (0)234 32 - 14101

A state-feedback approach to event-based control

Dipl.-Ing. Daniel Lehmann
lehmann@atp.rub.de

1 Introduction

Event-based control is a means to reduce the communication between the sensors, the controller and the actuators in a control loop by invoking a communication among these components only after an event has indicated that the control error exceeds a tolerable bound. This working principle differs fundamentally from that of the usual feedback loop, in which data are communicated from the sensor to the controller and from the controller to the actuator continuously or at every sampling instance given by a clock. Hence, in the control schemes currently used a communication takes place independently of the size of the control error and, in particular, also in case of small control errors when no information feedback is necessary to satisfy performance requirements on the plant. In these situations, the communication resources are used unnecessarily [1, 3].

2 Event-based control loop

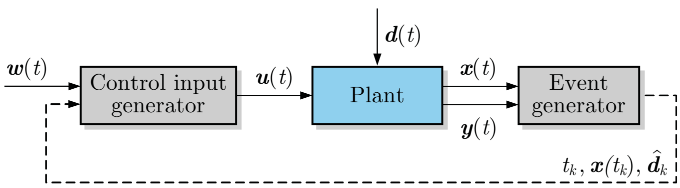

Fig. 1: Event-based control loop

The structure of the event-based control loop is depicted in Fig. 1, where the dashed arrows indicate that only at event times t_k information is communicated from the event generator towards the control input generator. Contrary, the arrows shown as solid lines are used continuously. The underlying consideration for investigating this structure is that feedback control, as opposed to feedforward control, is necessary in three situations [2]:

- An unstable plant has to be stabilised.
- Feedback should allow the controller to deal with model uncertainties.
- Unknown disturbances have to be attenuated.

To concentrate the investigations on a single item, it is assumed that the plant is asymptotically stable and no model uncertainties occur, and the only reason to communicate information via the dashed arrows in Fig. 1 is given by the situation that the disturbance $\boldsymbol{d}$ has an intolerable effect on the control output $\boldsymbol{y}$. The **plant** is considered to be linear and asymptotically stable, and the state $\boldsymbol{x}(t)$ is assumed to be measurable

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t). \end{aligned}$$

The **control input generator**, incorporating the function of the controller, determines the function $\boldsymbol{u}(t)$ for the time interval $t \in [t_k, t_{k+1})$ in dependence upon the information obtained at time t_k, where $\boldsymbol{u}(t)$ is generated by the following model:

$$\begin{aligned} \dot{\boldsymbol{x}}_{\mathrm{s}}(t) &= \bar{\boldsymbol{A}}\boldsymbol{x}_{\mathrm{s}}(t) + \boldsymbol{E}\hat{\boldsymbol{d}}_k, \quad t_k \le t < t_{k+1} \\ \boldsymbol{x}_{\mathrm{s}}(t_k) &= \boldsymbol{x}(t_k) \\ \boldsymbol{u}(t) &= -\boldsymbol{K}\boldsymbol{x}_{\mathrm{s}}(t), \end{aligned}$$

where $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$. Hence, in the time intervals between consecutive events, the control is carried out in an open-loop fashion.

The **event generator** determines the time instants t_k ($k = 0, 1, ...$) at which the next communication between the event generator and the control input generator is invoked by comparing the measured state $\boldsymbol{x}$ and the state $\boldsymbol{x}_{\mathrm{s}}$ determined by the control input generator, where events are generated if their distance is reaches a given threshold

$$d(\boldsymbol{x}(t_{k+1}), \boldsymbol{x}_{\mathrm{s}}(t_{k+1})) = \bar{e}.$$

The event generator additionally estimates the unknown disturbance

$$\hat{\boldsymbol{d}}_{k+1} = \hat{\boldsymbol{d}}_k + \left(\boldsymbol{A}^{-1}\left(\mathrm{e}^{\boldsymbol{A}(t_{k+1}-t_k)} - \boldsymbol{I}\right)\boldsymbol{E}\right)^{+} (\boldsymbol{x}(t_{k+1}) - \boldsymbol{x}_{\mathrm{s}}(t_{k+1}))$$

and communicates the state and the disturbance estimate at event times towards the control input generator.

The **state trajectory** $\boldsymbol{x}(t)$ of the event-based closed-loop system can be decomposed into two parts (Fig. 2)

$$\begin{aligned} \boldsymbol{x}(t) &= \left.\mathrm{e}^{\bar{\boldsymbol{A}}(t-t_\ell)}\boldsymbol{x}(t_\ell) + \bar{\boldsymbol{A}}^{-1}\left(\mathrm{e}^{\bar{\boldsymbol{A}}(t-t_\ell)} - \boldsymbol{I}\right)\boldsymbol{E}\hat{\boldsymbol{d}}_\ell \ \right\} \boldsymbol{x}_{\mathrm{s}} \\ &\quad + \left.\int_{t_\ell}^{t} \mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{E}\left(\boldsymbol{d}(\tau) - \hat{\boldsymbol{d}}_\ell\right) \right\} \boldsymbol{x}_\Delta. \end{aligned}$$

The state trajectory $\boldsymbol{x}_{\mathrm{s}}(t)$ shows the effect of the constant disturbance $\boldsymbol{d}(k) = \hat{\boldsymbol{d}}_\ell$ on a state-feedback control loop for the

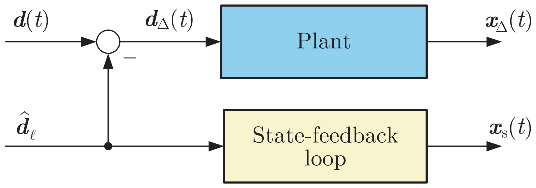

Fig. 2: Decomposition of $\boldsymbol{x}(t)$

interval $k \in \{t_\ell, \ldots, (t_{\ell+1})\}$ which has the same effect in the event-based control system and in the state-feedback system. The difference state $\boldsymbol{x}_\Delta(t)$ describes the difference between the state-feedback loop and the event-based control system for time-varying disturbances, where the difference $\boldsymbol{d}(t) - \hat{\boldsymbol{d}}_\ell$ affects the (uncontrolled) plant.

3 Application

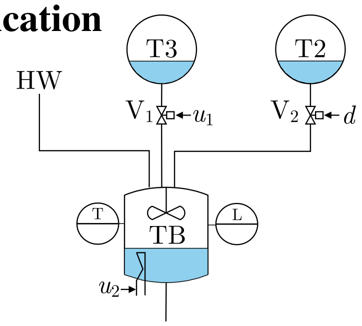

Fig. 3: Thermofluid process

The event-based strategy is applied to the thermofluid process depicted in Fig 3. The process consists of the cylindrical batch reactor TB, which is connected with two spherical tanks T3 and T2 by pipes. An additional water supply HW feeds the reactor TB with a constant inflow. The inflow into TB from T3 and T2 can be controlled by means of the valves V_1, V_2. The heating can be used to increase the temperature of a fluid in the reactor TB. The level l_{TB} and the temperature ϑ_{TB} of the fluid in the reactor TB can be measured.

The valve angle u_1 of valve V_1 as well as the power u_2 of the heating rods are used as inputs $\boldsymbol{u}(t) = (u_1(t) \;\; u_2(t))^{\mathrm{T}}$, whereas the valve angle d of valve V_2 is used in order to realize desired disturbance characteristics. The behaviour of the event-based control loop subject to exogenous disturbances is depicted in Fig. 4.

In the first investigation (left-hand side of Fig. 4) the plant is subject to a constant disturbance $d(t) = \bar{d}$. An event takes place at time t_1, where

$$\|\boldsymbol{x}_\Delta(t_1)\|_\infty = |x_1(t_1) - x_{s1}(t_1)| = \bar{e}$$

holds (cf. second subplot from top). At this event time the disturbance magnitude $\bar{d}$ is correctly estimated by the disturbance estimator ($\hat{d}_1 = \bar{d}$) and communicated to the control input generator together with the state information $\boldsymbol{x}(t_1)$. At steady state, both $\boldsymbol{x}(t)$ (solid) and $\boldsymbol{x}_s(t)$ (dashed) coincide and behave like the continuous state-feedback loop the behaviour of which is shown by the dotted line. No further event occurs. The steady-state control error occurs due to the fact that a state feedback is a proportional controller. It can be avoided by using controllers with integral action.

In the second investigation (right-hand side of Fig. 4) the disturbance magnitude changes after the first events as shown by the dotted line in the top subplot. Five events take place until the disturbance remains constant and its magnitude is estimated with sufficient accuracy. The events are generated, because the difference state $\boldsymbol{x}_\Delta(t)$ satisfies the equality $|x_{\Delta,1}(t_k)| = \bar{e}$, $k = 1, 2, 3, 4$. At time t_1 the estimate $\hat{d}_1$ of the disturbance $d(t)$ in the preceding time-interval $[0, t_1)$ is communicated to the control input generator. As the disturbance varies, the estimate $\hat{d}_1$ and the disturbance $d(t)$ differ for $t \geq t_1$ and a new event occurs at time t_2. The new disturbance estimate $\hat{d}_2$ determined at this event time describes a weighted average of $d(t)$ of the preceding time interval $[t_1, t_2)$. After event time t_4 the disturbance is constant and no further event is generated because the difference $\bar{d}(t) - \hat{d}_4$ is sufficiently small. No further communication is invoked and the relation $\boldsymbol{x}(\infty) - \boldsymbol{x}_s(\infty) \neq \boldsymbol{0}$ holds as depicted by the two middle subplots on the right-hand side of the figure.

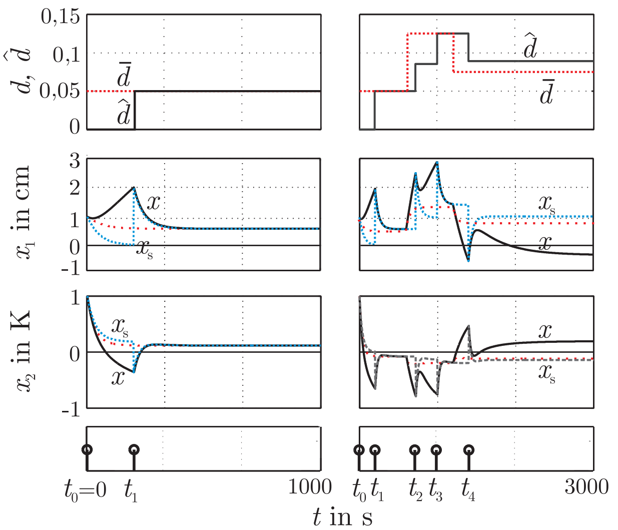

Fig. 4: Behaviour of the process model subject to two different exogenous disturbances

4 Cooperation

This research is supported by the *Deutsche Forschungsgemeinschaft* (Schwerpunktprogramm *Regelungstheorie digital vernetzter dynamischer Systeme*) and is being carried out in cooperation with the Mathematical Institute at Universität Bayreuth and the Mathematical Institute at Technische Universität München.

References

[1] D. Lehmann and J. Lunze. Event-based control: A state feedback approach. In *Proc. 10th European Control Conference*, Budapest, Hungary, 2009. Accepted.

[2] J. Lunze. *Regelungstechnik 1*. Springer-Verlag, Heidelberg, fünfte edition, 2005.

[3] J. Lunze and D. Lehmann. A state-feedback approach to event-based control. *Automatica*. submitted.