Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150 Phone +49 - (0)234 32 - 28071
D-44805 Bochum, Germany Fax +49 - (0)234 32 - 14101

On-line local reconstruction of optimal switching surfaces for periodically operated DCCS

Dipl.-Ing. Axel Schild
schild@atp.rub.de

1 Introduction

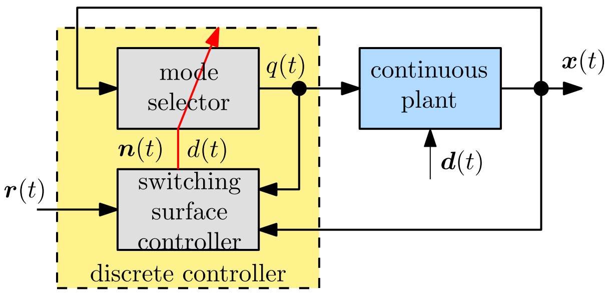

Figure 1: Structure of a discretely controlled system.

Discretely controlled continuous systems (DCCS) represent an important class of hybrid systems with many applications in power electronics, process engineering and robotics. They appear as a control loop of a continuous plant and a discrete-event controller (Fig. 1). The plant dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \qquad (1)$$

depend on the activated *mode of operation* $q(t) \in \mathcal{Q}$ and are influenced by disturbances $\boldsymbol{d}(t)$. The *discrete controller* implements a state-dependent switching law to trigger mode transitions in the plant, such that specifications $\boldsymbol{r}(t)$ imposed on the continuous state $\boldsymbol{x}(t)$ are met.

A central control task of DCCS concerns the stabilization of stationary periodic operations, which map into p-periodic limit cycles Γ in the state space (see Fig. 2). Each limit cycle can be characterized by its periodic mode sequence $Q_\Gamma = (\bar{q}_0^\star \dots \bar{q}_{p-1}^\star)$ and the associated switch points $\mathcal{X}_\Gamma = \{\bar{\boldsymbol{x}}^\star(\bar{q}_0^\star), \dots, \bar{\boldsymbol{x}}^\star(\bar{q}_{p-1}^\star)\}$. In general, orbital stability must be achieved by timing mode transitions properly, as the mode order is fixed through Γ.

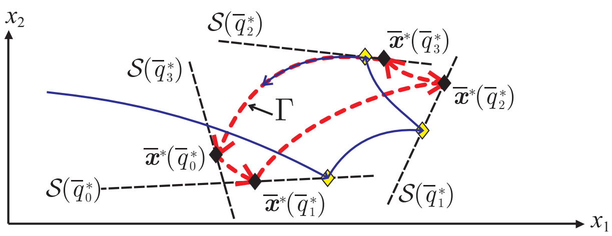

Figure 2: Stabilizing switching planes for a limit cycle Γ.

2 Project goals

State-based switching laws define a switching surface configuration $\left\{\mathcal{S}\left(\bar{q}_k^\star\right)\right\}_{k=0}^{p-1}$ in the state space (Fig. 2). A transition from $\bar{q}_k^\star$ to $\bar{q}_{k+1}^\star$ is triggered, whenever the state trajectory $\boldsymbol{x}(t)$ intersects with the surface $\mathcal{S}\left(\bar{q}_k^\star\right)$ at the switch point $\bar{\boldsymbol{x}}(k)$. Due to implementation issues, simple hyperplanes

$$\mathcal{T}(\bar{q}_k^\star) = \left\{\boldsymbol{x} \mid \boldsymbol{n}^{\mathrm{T}}(q)\,\boldsymbol{x} - d(q) = 0\right\}$$

are typically chosen as switching surfaces $\mathcal{S}\left(\bar{q}_k^\star\right)$. In [4], a model-based design procedure for switching planes was proposed on the basis of a linearized model, which establishes desired loop properties like

1. local orbital stability of Γ,
2. a desired transient loop behavior and
3. robustness

at least inside a local environment $\mathcal{B}_\epsilon(\Gamma)$ of the reference orbit Γ. Unfortunately, due to the linearization, ϵ may be arbitrarily small. This fact provides the motivation of this research, which focuses on a novel adaptive control concept with the **objective** to enlarge the region of attraction of Γ significantly.

3 Adaptive control approach

To meet the project goal, an adaptive control approach, which combines the design of locally optimal switching planes [4] with a switching surface controller (SSC) [3], is pursued. As shown in Fig. 1, the proposed loop structure consists of two cascaded loops that operate on different time scales as well as different regions of validity. The inner loop, which consists of the mode selector and the plant, has the primary task of maintaining the overall functionality of the DCCS, i.e. the conservation of switching. To this end, the mode selector implements an explicit switching law, which detects the instant

$$\bar{t}(k+1) = \min_{t \geq \bar{t}(k)} t \ :\ \left|\boldsymbol{n}^{\mathrm{T}}(t)\boldsymbol{x}(t) - d(t)\right| \geq 0$$

of the subsequent switching event $\bar{q}_k^\star \to \bar{q}_{k+1}^\star$ by means of evaluating a parametrized switching condition.

The externally provided parameters $\boldsymbol{n}(t)$, the plane normal, and the plane offset $d(t)$, are incrementally updated by the switching surface controller. This adaptation is executed repeatedly at equidistant sample times $t_s = s \cdot \Delta t$. In between two consecutive updates, the inner loop continuously checks for the occurrence of the next event. Thus, even large Δt only result in a performance degradation, but rarely in a complete failure of the overall loop. This property is a major advantage over existing control approaches. Moreover, the inner loop alone is capable of compensating disturbances $\boldsymbol{d}(t)$ of moderate amplitude.

As the mode selector constitutes a trivial *explicit* component of the controller, the **main task** of this project is to derive and efficiently evaluate the *implicit* SSC update law, which requires the repeated solution of an MPC-like *crawling-window optimal control problem* in real time. A central aspect, here, concerns the efficient implementation of the switching surface controller to keep the computational burden as low as possible and allow at best for an almost continuous-time adjustment of the planes.

4 Iterative plane adjustment

The proposed switching surface controller builds upon ideas of model predictive control and, on an abstract level, constitutes a dynamic nonlinear discrete-time element

$$\boldsymbol{t}_p(t_{s+1}) = \boldsymbol{f}_c\left(\boldsymbol{t}_p(t_s), \boldsymbol{x}(t_s), q(t_s), \Gamma\right), \quad \boldsymbol{t}_p(0) = \boldsymbol{t}_{p,0} \tag{2}$$

$$\boldsymbol{n}(t_s) = \boldsymbol{h}_n\left(\boldsymbol{t}_p(t_s), \boldsymbol{x}(t_s), q(t_s), \Gamma\right) \tag{3}$$

$$d(t_s) = \boldsymbol{h}_d\left(\boldsymbol{t}_p(t_s), \boldsymbol{x}(t_s), q(t_s), \Gamma\right) \tag{4}$$

that processes the input history $\boldsymbol{x}(\cdot)$ and $q(\cdot)$, the hybrid state trajectory of the inner loop, and the specifications Γ into suitable parameter trajectories $\boldsymbol{n}(\cdot)$ and $d(\cdot)$. The elements of the internal state vector $\boldsymbol{t}_p(t_s) = \{\bar{t}(j)\}_{j=k(t_s)+1}^{k(t_s)+N(t_s)+1}$ represent the *predicted* switching times for the *next* $N(t_s)$ mode transitions, where $k(t_s)$ enumerates the number of executed switchings until time t_s. The initial value $\boldsymbol{t}_{p,0}$ characterizes the initial switching plane configuration. When t_s corresponds to a switching time $\bar{t}(k)$, the state $\boldsymbol{t}_p(t_s)$ must be re-initialized appropriately.

All functions $\boldsymbol{f}_c$, $\boldsymbol{h}_n$ and $\boldsymbol{h}_d$ are highly nonlinear and $\boldsymbol{f}_c$, in particular, cannot be stated in closed form. An evaluation of $\boldsymbol{f}_c$ requires the execution of a single *Newton iteration* for the nonlinear optimal control problem

$$\min_{\boldsymbol{t}_p(t)} \sum_{j=k}^{k+N(t)} \left(\int_{\bar{t}(j)}^{\bar{t}(j+1)} L_j(\boldsymbol{x}(t))\mathrm{d}t + \phi_k(\bar{\boldsymbol{x}}(j)) \right) + \phi_T(\bar{\boldsymbol{x}}(k+N(t)+1)) \tag{5}$$

$$\text{subject to dynamics (1)} \tag{6}$$

$$\bar{t}(j) \leq \bar{t}(j+1) \tag{7}$$

$$\psi(\bar{\boldsymbol{x}}(k+N(t)+1) \leq 0 \ , \tag{8}$$

which is cast over a finite *prediction horizon* including the next $N(t)$ mode transitions. The instantaneous costs L_j assess the transient performance, while ϕ_j account for switching costs. Besides smoothness, the functions can be chosen nearly arbitrarily by the designer. The terminal costs ϕ_T and the terminal constraints ψ, on the other hand, must be properly chosen in order to ensure stability of the limit cycle.

As it was shown in [1, 2] problem (5)-(8) can be translated into an equivalent classical discrete-time optimal control problem by means of Lebesgue sampling. A Newton step for this equivalent problem can then be determined via standard nonlinear programming techniques. Besides the updated state value $\boldsymbol{t}_p(t_{s+1})$, which constitutes an improved estimate of the true optimal switching times, appropriate values $\boldsymbol{n}(t_s)$ and $d(t_s)$ are obtained as a by-product with negligible costs by exploiting the concept of *neighboring extremals*.

Under reasonable assumptions, the state $\boldsymbol{t}_p(t), t > \bar{t}(j)$ quickly converges to a stationary value after re-initialization at $\bar{t}(j)$ (see Fig. 3 and 4). If convergence is ensured before the next mode transition is triggered, the overall loop performance is MPC-optimal and stability of Γ is attained for a typically large subset of the state space.

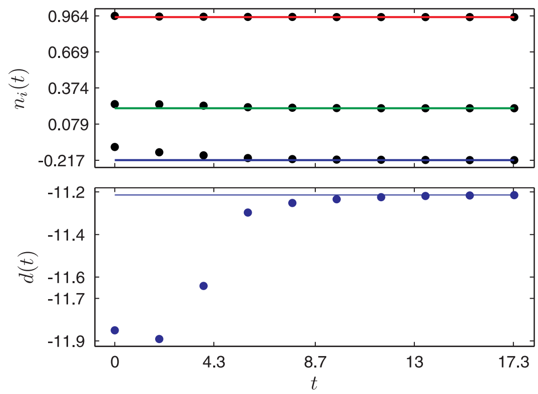

Figure 3: Parameter trajectories quickly converging towards optimal values (solid lines) before the next mode transition is triggered.

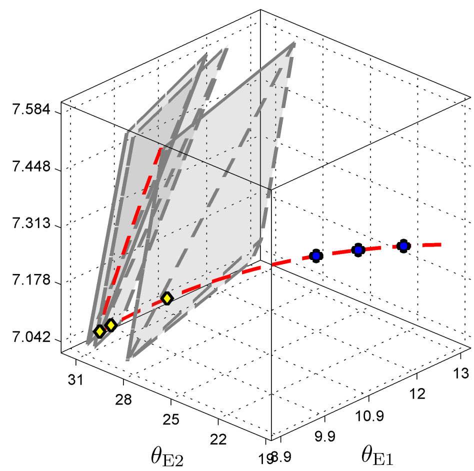

Figure 4: Dynamically adapted switching plane in state space.

References

[1] X. Ding, A. Schild, M. Egerstedt, and J. Lunze. Real-time optimal feedback control of switched autonomous systems. In *Proceedings of the IFAC ADHS*, 2009 (submitted).

[2] A. Schild, X. Ding, M. Egerstedt, and J. Lunze. Design of optimal switching surfaces for switched autonomous systems. In *Proc. of 42nd IEEE CDC*, 2009 (submitted).

[3] A. Schild and J. Lunze. Stabilization of limit cycles of discretely controlled continuous systems by controlling switching surfaces. In *Hybrid Systems: Computation and Control (HSCC 2007)*, volume 4416 of *LNCS*, pages 515–528, 2007.

[4] A. Schild and J. Lunze. Switching surface design for periodically operated discretely controlled continuous systems. In *Hybrid System: Computation and Control*, pages 471–485. Springer Verlag, Heidelberg, 2008.