Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150 Phone +49 - (0)234 32 - 28071
D-44805 Bochum, Germany Fax +49 - (0)234 32 - 14101

Reconfigurable Control of Piecewise Affine Systems

Dipl.-Ing. Jan Richter
richter@atp.rub.de

1 Introduction

The task of control reconfiguration is to automatically find a new controller after the occurrence of a fault or failure, such that the reconfigured closed-loop system approximately satisfies the same control specifications as the nominal closed-loop system [1, 2]. The synthesis must happen autonomously and unsupervised by human engineers, which distinguishes the reconfiguration task from ordinary control synthesis. In this project, the fault-hiding concept is extended from linear systems to PWA systems for actuator faults and sensor faults alike.

2 Reconfigurable control problem

In this project, the control reconfiguration problem is investigated for piecewise affine (PWA) systems. PWA models express state-dependent switching dynamics. Furthermore, they may be used to approximate continuous nonlinear dynamics. This class of systems is described by the equations

$$\Sigma_P : \begin{cases} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{b}_i + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B}_{\boldsymbol{d}}\boldsymbol{d}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ & \quad \text{for } \boldsymbol{x} \in \Lambda_i,\ i = 1, \dots, p \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \\ \boldsymbol{z}(t) &= \boldsymbol{C}_{\boldsymbol{z}}\boldsymbol{x}(t), \end{cases} \tag{1}$$

where it is assumed that the output is a linear combination of the states and the right-hand side is continuous in $\boldsymbol{x}$, $\boldsymbol{u}$ and $\boldsymbol{d}$, and $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u} \in \mathbb{R}^m$, $\boldsymbol{d} \in \mathbb{R}^n$. The polytopes Λ_i represent the state-space partition and define the switching law. Actuator and sensor faults are modelled as changes in the vector field,

$$\Sigma_{Pf} : \begin{cases} \dot{\boldsymbol{x}}_f(t) &= \boldsymbol{A}_i \boldsymbol{x}_f(t) + \boldsymbol{b}_{f,i} + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B}_{\boldsymbol{d}}\boldsymbol{d}(t) \\ & \quad \text{for } \boldsymbol{x}_f \in \Lambda_i,\ i = 1, \dots, p,\ \boldsymbol{x}_f(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}_f(t) &= \boldsymbol{C}_f\boldsymbol{x}_f(t) \\ \boldsymbol{z}_f(t) &= \boldsymbol{C}_{\boldsymbol{z}}\boldsymbol{x}_f(t), \end{cases} \tag{2}$$

where the index f is used to denote faulty behaviour. This fault model captures

- actuator degradation and failure as well as blockage at arbitrary positions,
- sensor degradation and failure.

Control reconfiguration should solve the following problems

- *closed-loop stability recovery*,
- *setpoint tracking recovery*.

3 Reconfiguration approach

The central idea used in this project is to retain the nominal controller Σ_C as a part of the closed-loop system to preserve the design knowledge it contains, or to minimize the required training effort in the case where the controller is a human being. The *fault-hiding principle* enables this idea by inserting a reconfiguration block Σ_R into the closed-loop system.
The reconfiguration block (Figure 1) is realised for actuator and sensor faults by the combination of a *PWA virtual sensor*

$$\Sigma_S : \begin{cases} \dot{\hat{\boldsymbol{x}}}_f(t) &= \boldsymbol{A}_{\delta,i}\hat{\boldsymbol{x}}_f(t) + \boldsymbol{a}_{fi} + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{L}\boldsymbol{y}_f(t) \\ & \quad \text{for } \hat{\boldsymbol{x}}_f \in \Lambda_i,\ i = 1, \dots, p,\ \hat{\boldsymbol{x}}_f(0) = \hat{\boldsymbol{x}}_{f0} \\ \hat{\boldsymbol{y}}_c(t) &= \boldsymbol{y}_f(t) + (\boldsymbol{C} - \boldsymbol{C}_f)\hat{\boldsymbol{x}}_f(t) \\ \boldsymbol{A}_{\delta,i} &\triangleq \boldsymbol{A}_i - \boldsymbol{L}\boldsymbol{C}_f. \end{cases} \tag{3}$$

and a *PWA virtual actuator*

$$\Sigma_A : \begin{cases} \dot{\tilde{\boldsymbol{x}}}(t) &= \boldsymbol{A}_j\tilde{\boldsymbol{x}}(t) + \boldsymbol{a}_j + \boldsymbol{B}\boldsymbol{u}_c(t),\ \tilde{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_{f0} \\ & \quad \text{for } \tilde{\boldsymbol{x}} \in \Lambda_j,\ j = 1, \dots, p \\ \boldsymbol{y}_c(t) &= \hat{\boldsymbol{y}}_c(t) + \boldsymbol{C}\boldsymbol{x}_\Delta(t) \\ \boldsymbol{u}_f(t) &= \boldsymbol{M}\boldsymbol{x}_\Delta(t) + \boldsymbol{B}_f^+\boldsymbol{a}_{\Delta j} \end{cases} \tag{4}$$

where

$$\boldsymbol{x}_\Delta(t) \triangleq \tilde{\boldsymbol{x}}(t) - \hat{\boldsymbol{x}}_f(t),\ \boldsymbol{e}(t) \triangleq \hat{\boldsymbol{x}}_f(t) - \boldsymbol{x}_f(t),\ \boldsymbol{a}_{\Delta j} \triangleq \boldsymbol{a}_j - \boldsymbol{a}_{fj}.$$

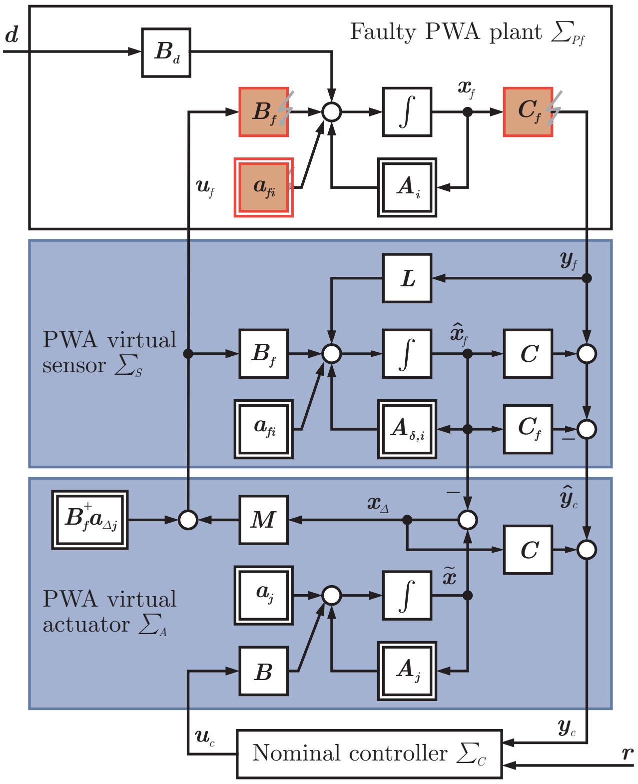

Figure 1: PWA reconfiguration block and closed-loop system.

The virtual actuator is a predictor for the difference between the state trajectories of the nominal and faulty plant starting from the same initial state. The prediction is used for state feedback, which achieves the required closed-loop stability. The term $\boldsymbol{B}_f^+\boldsymbol{a}_{\Delta j}$ compensates blocked actuators if and only if

$$\boldsymbol{a}_{\Delta j} \in \operatorname{im} \boldsymbol{B}_f$$

and if the actuation ranges of the remaining actuators are sufficiently large.

4 Stability, performance and synthesis

It has been shown [3, 4] that

- the reconfigured closed-loop system (2), (3), (4) is globally input-to-state stable (ISS)

if the sufficient conditions for the observer error ISS

$$\boldsymbol{X}_s = \boldsymbol{X}_s^T \succ 0$$
$$\boldsymbol{X}_s\boldsymbol{A}_i + \boldsymbol{A}_i^T\boldsymbol{X}_s - \boldsymbol{Y}_s\boldsymbol{C}_f - \boldsymbol{C}_f^T\boldsymbol{Y}_s^T \prec 0,\ i = 1,\ldots,p$$

and for the difference system ISS

$$\boldsymbol{X}_a = \boldsymbol{X}_a^T \succ 0$$
$$\boldsymbol{A}_i\boldsymbol{X}_a + \boldsymbol{X}_a\boldsymbol{A}_i^T - \boldsymbol{B}_f\boldsymbol{Y}_a - \boldsymbol{Y}_a^T\boldsymbol{B}_f^T \prec 0,\ i = 1,\ldots,p$$

are satisfied, where the gains $\boldsymbol{L} \triangleq \boldsymbol{X}_s^{-1}\boldsymbol{Y}_s$ and $\boldsymbol{M} \triangleq \boldsymbol{Y}_a\boldsymbol{X}_a^{-1}$ are determined by the stability requirement. Further work has

- extended the approach towards setpoint tracking recovery for constant setpoints and constant disturbances [3, 5], and
- shown that this approach is robust against uncertainty of the model of the faulty plant [3].

The tracking extensions are based on internal models of the exogenous reference and disturbance signals, which are embedded into the reconfiguration block.

5 Example

A two-tank system as shown in Figure 2 illustrates these ideas. The right tank T_2 represents a direct supply to a consumer, hence its level must meet strict requirements to produce the desired supply pressure. The left tank T_1 is a buffer fed by a variable speed pump. The connecting flow between the tanks is controlled by continuous control valve u_L.

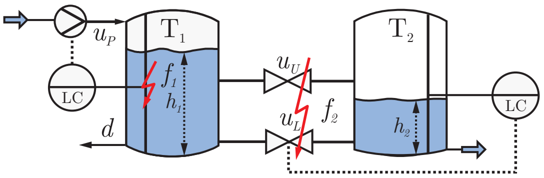

Figure 2: Laboratory two-tank system.

An example of reconfigurable control after failure f_1 of the level sensor for h_1 at time 35 s, blockage of the lower valve u_L, and degradation of the upper valve u_U (f_2 at time 20 s) is shown in Figure 3. Note that the plant is represented by full nonlinear dynamics, whereas the reconfiguration block is based on a piecewise affine model. In spite of the model mismatch, the closed-loop system robustly recovers stability and tracking.

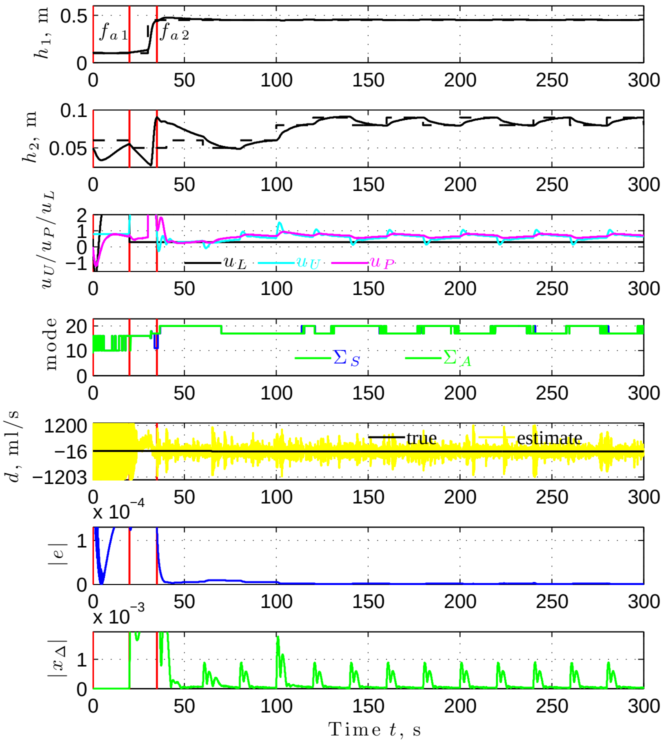

Figure 3: Control reconfiguration after sensor and valve faults.

The disturbance estimate (yellow) also reflects the model error, but its mean is close to the true disturbance value. The approach has also been evaluated by means of a thermofluid process [3, 6].

6 Cooperation

This project is done in cooperation with Prof. M. Heemels and Prof. N. van de Wouw from the Control Systems Technology Group (CST) at Eindhoven University of Technology (TU/e), The Netherlands.

References

[1] M. Blanke, M. Kinnaert, J. Lunze, and J. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Springer Verlag, Heidelberg, 2nd edition, 2006.

[2] J. Lunze and J. H. Richter. Reconfigurable fault-tolerant control: a tutorial introduction. *European J. Control*, 14(5):359–386, 2008.

[3] J. H. Richter. *Reconfigurable Control of Nonlinear Dynamical Systems – a fault-hiding approach*. PhD thesis, Fakultät für Elektrotechnik und Informationstechnik, Ruhr-Universität Bochum, 2009. Submitted.

[4] J. H. Richter, W. P. M. H. Heemels, N. van de Wouw, and J. Lunze. Reconfigurable control of PWA systems with actuator and sensor faults: stability. In *Proceedings of the 47th Conf. Decision and Control (CDC)*, pages 1060–65, Cancun, Mexico, December 2008.

[5] J. H. Richter, W. P. M. H. Heemels, N. van de Wouw, and J. Lunze. Reconfigurable control of PWA systems with actuator and sensor faults: stability and tracking, 2009. Submitted.

[6] S. Adelt. Rekonfigurierbare Regelung stückweise affiner Systeme: Modellbildung und Experimente. Diplomarbeit, Lehrstuhl für Automatisierungstechnik und Prozessinformatik, Fakultät für Elektrotechnik und Informationstechnik, Ruhr-Universität Bochum, March 2009. ATP 0064.