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Hybrid Event-Based Control Systems

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1 Hybrid event-based control

In this project hybrid event-based control (HEBC) systems are investigated, where a continuous control loop is extended by an event-based controller (Figure 1). The plant G has two input signals u_1, u_2. The control input u_1 is adjusted continuously by the controller K' in order to attenuate disturbances and force the plant to follow a command signal. The second control input u_2 is switched based on discrete events, shifting the operation point of the plant.

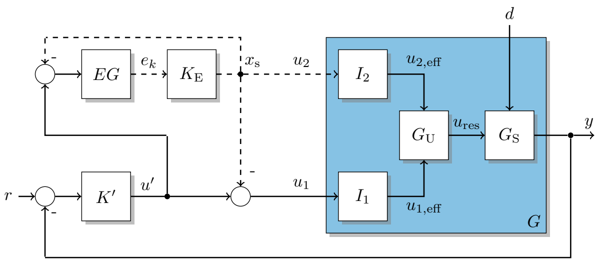

Figure 1: Structure of HEBC systems

The motivation for hybrid event-based control systems comes from different applications in which continuous and event-based control principles are naturally combined. Examples involve automotive systems, energy supply systems and chemical or biological process plants. Typically the event-based switching of the control input u_2 is necessary to prevent control input u_1 from exceeding its limitations. Depending on the application, these limitations may hold for all times $t \geq 0$ or only in a steady state for $t \to \infty$.

HEBC systems can be classified by the way in which the input signals u_1 and u_2 interact:

- In HEBC systems with the **multiplicative interaction** $u_{\mathrm{res}} = u_{1,\mathrm{eff}} \cdot u_{2,\mathrm{eff}}$, the event-based controller changes the gain of the continuous control loop. Examples include the speed control of vehicles with automatic gearboxes and the terminal voltage control of electrical power plants.
- In HEBC systems with an **additive superposition** $u_{\mathrm{res}} = u_{1,\mathrm{eff}} + u_{2,\mathrm{eff}}$ the two signals u_1 and u_2 affect the plant in an equal manner. Examples for this structure can be found in hybrid positioning systems or hybrid drive trains as well as chemical or biological process plants.

The combination of event-based switching and continuous adjustment of the control input signals leads to a closed-loop system with hybrid dynamics [1]. The class of hybrid dynamical systems is characterized by an interconnection of continuous and discrete dynamics giving rise to several problems with regard to stability and existence of Zeno behaviour. Nevertheless, in many applications the hybrid character of the system is ignored and the controllers are tuned heuristically.

2 Project aims

The aim of the project is to develop methods for the design of the controller and the analysis of the resulting closed-loop system, especially taking into account the hybrid character of the system.

The difficulty of the control design problem results from the constraints of the control inputs. On the one hand, for the continuous input u_1 a continuous controller can be designed by methods from linear control theory, but the resulting controller cannot be implemented because the magnitude of the input is restricted and, hence, the controller has to utilise the second control input signal in order to shift the operation point of the system.

The main approach followed in this project is characterised by splitting up the hybrid controller into a purely continuous reference controller K' and an event-based path consisting of the event generator EG and the event-based controller K_{E}.

3 HEBC systems with linear components

The plant G consists of several subsystems (Fig. 1). The subsystems I_1 and I_2 represent the input dynamics of u_1 and u_2 respectively. The effective control input signals $u_{1,\mathrm{eff}}$ and $u_{2,\mathrm{eff}}$ are combined to the resulting control input signal u_{res} by the subsystem G_{U}. The actual plant dynamics G_{S} are affected by u_{res} and the disturbance signal d.

Basic properties of HEBC systems have been presented in [2] and [3], where it is assumed that the subsystems of the plant G behave linearly and the control input signals add up to a resulting control input signal u_{res} (additive superposition). In this case the hybrid closed-loop system can be approximated by a *linear approximate system* consisting of the linear closed-loop (K', I_2, G_{S}) and a bounded input signal d_{U}.

Figure 3 shows a simulation of an HEBC system subject to a jump of the reference signal at $t = 5$ and a jump of

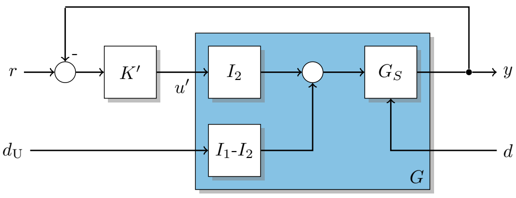

Figure 2: Linear approximate system

the disturbance signal at $t = 75$. Whenever the continuous control input signal $u_1(t)$ reaches the threshold $\bar{e} = 1$, the discrete control input signal $u_2(t)$ is switched to a new value and consequently u_1 is reset to zero. The switching

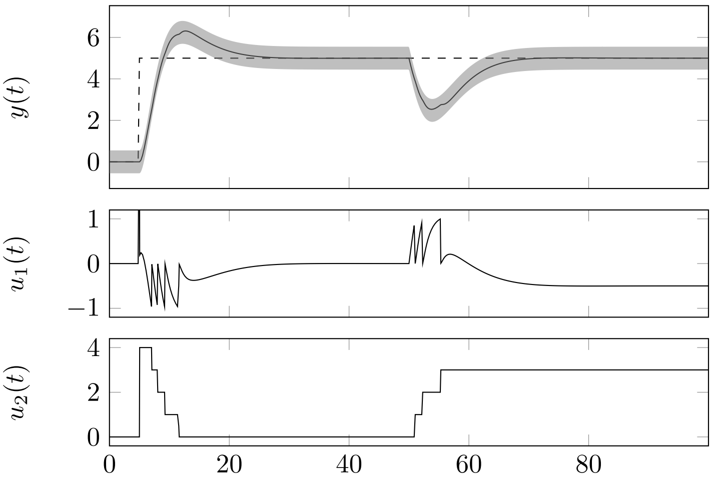

Figure 3: Simulation of an HEBC system

of the control input signals causes a disturbance of the closed-loop system. However, the effect of this disturbance is bounded. An upper bound of the difference between the output $y(t)$ of the hybrid system and the output $\hat{y}(t)$ of the linear approximate system can be derived. The grey band in the top diagram of Figure 3 marks the predicted trajectory of $y(t)$.

Based on the linear approximation, methods from linear control theory can be applied to design the controller and prove stability of the system. Further conditions on the reference controller and the event threshold can be derived ensuring that the system asymptotically tracks step-wise reference signals and does not admit Zeno behaviour.

4 Extensions of the hybrid control structure

In many applications the input dynamics I_1, I_2 differ considerably from each other. If I_1 is fast but I_2 is slow, the reference controller K' has to be designed considering the slow dynamics of I_2. Therefore, the controller cannot utilise the potential performance of I_1 regarding transient response behaviour. This is a major drawback of the design method presented in [2].

An approach to overcome this drawback is the introduction of an input generator system L in the event-based path of the control system. The task of the input generator is to "shape" the signals that are subtracted from $u_1(t)$ and added to $u_2(t)$ after a discrete event such that the effect of the event-based switches on the resulting control input signal $u_{\text{res}}(t)$ is suppressed. Figure 4 shows the structure of the HEBC system extended by the input generator system L.

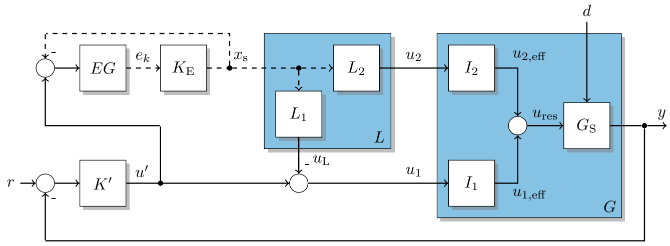

Figure 4: Structure of the extended HEBC system

Another extension of the hybrid control structure that should be investigated in this project is a dynamical event-generator EG. An event-generator with internal dynamics enables the event-based controller to react not only to the magnitude of the control signals but also to derivatives or integrals of these signals. It therefore promises advantages over the rather simple event-generator used in [2] and [3].

5 HEBC systems with multiplicative interaction

The previous considerations are based on HEBC systems with additive control input signals. However, in many applications the control input signals interact in a multiplicative manner (e.g. control systems where a gear ratio or a transformation ratio is switched). In this case the subsystem G_{U} is a multiplication and, hence, the plant G has nonlinear dynamics.

The nonlinear HEBC system can be modelled by a switched system, where the switching of the control input signal $u_2(t)$ changes the gain of the continuous control input signal $u_1(t)$. In order to extend the methods and results to this class of systems the continuous reference controller should be switched to a new suitable controller whenever the value of $u_2(t)$ is changed.

The switching between different reference controllers K' strengthens the hybrid character of the resulting closed-loop system. Hence, the typical problems of hybrid systems (stability, Zeno behaviour, inter-event times etc.) have to be analysed carefully.

References

[1] J. Lunze, F. Lamnabhi-Lagarrigue (Eds.) *Handbook of Hybrid Systems Control*, Cambridge University Press, 2009.

[2] T. Noesselt, M. Schultalbers, J. Lunze *A design method for hybrid event-based control systems*, Preprints of the 5th IFAC Conf. on Analysis and Design of Hybrid Systems, 2015.

[3] T. Noesselt, M. Schultalbers, J. Lunze *Event-Separation Properties and Asymptotic Behaviour of Hybrid Event-Based Control Systems*, 19th ACM International Conference on Hybrid Systems: Computation and Control (HSCC), 2016, accepted.