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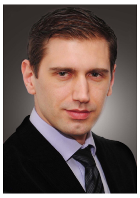

Synchronization of oscillators in complex networks

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1 Introduction

The achievement of a coherent behavior of a large number of autonomous systems (agents) is a common control objective in process automation systems. The angle synchronization of rollers in paper converting machines or periodic processes which occur in the production of chemicals or the coating of materials are only a few examples. The examples are not only restricted to technical systems. For understanding the behavior of biological systems like flocks of birds, shoals or flashing fireflies it is necessary to analyse the interconnection behavior.

The aim of this project is the development of new controller design methods for the synchronization of nonlinear oscillators which can be used for the modelling of a broad range of dynamical systems. The requirement of asymptotic synchronization

$$\lim_{k\to\infty} \|y_i(k) - y_j(k)\| = 0, \qquad i, j = 1, 2, \ldots, N \tag{1}$$

of the oscillators output signals y_i builds an uniform control objective which can only be reached if couplings among the overall system are introduced through which the input u_i of every agent is influenced by every other agent directly or indirectly. Hence, the synchronizing controller has to be chosen so as to create the required couplings. In this project the networked controller was picked to consist of two components, the local control algorithms R_i and the communication network $\boldsymbol{L}$ (Fig. 1).

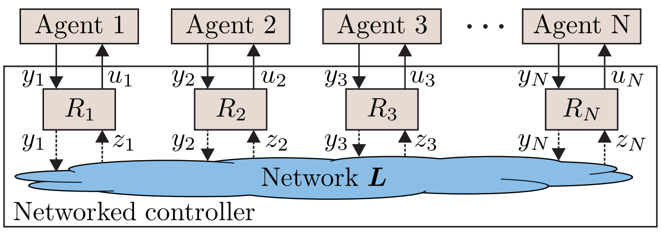

Figure 1: Networked multi-agent system

Typically, synchronization is considered for static or randomly changing network structures. Besides this types of networks there also exist wireless ad hoc networks, where information can only be exchanged by point-to-point connections. In this project it is assumed that each agent can only exchange information with one another agent at the same time. Hence, new control methods have to be developed for the synchronization of autonomous agents by a pairwise information exchange.

2 System model and network

The oscillators are modeled by the nonlinear state-space models

$$\Sigma_i : \begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}(\boldsymbol{x}_i(t), u_i(t)), \quad \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i} \\ y_i(t) = \boldsymbol{h}(\boldsymbol{x}_i(t)), \end{cases}$$

where $\boldsymbol{x}_i$ is the state vector, u_i the scalar control input and y_i the output signal of the i-th agent.

The communication network is described by a bidirectional graph and the associated Laplacian matrix $\boldsymbol{L}$, where the i-th and j-th agents are referred to as neighbors if $(\boldsymbol{L})_{ij} = 1$. In this project it is assumed that the communication network $\boldsymbol{L}$ has the following properties:

- Each agent can only be connected to one of its neighbors at the same time.
- Disjoint pairs of agents can be connected at the same time.
- The network $\boldsymbol{L}$ is instantaneous and lossless.

3 Control algorithm

Figure 1 illustrates the structure of the networked controller, where local control algorithms R_i together with the communication network $\boldsymbol{L}$ are used to couple the agents. The local control algorithms have the structure shown in Fig. 2. The idea behind the structure of the algorithm is

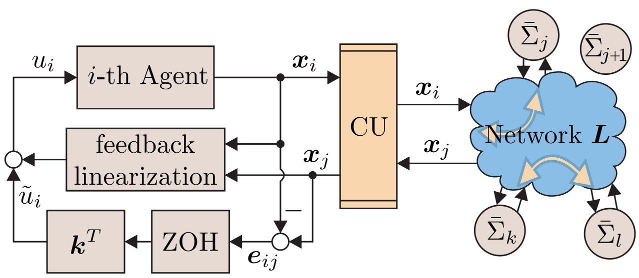

Figure 2: Structure of the control algorithms

threefold. First, the communication unit (CU) has to set up an information exchange to a neighboring agent which is currently not connected to one another agent. Secondly, the block feedback linearization is used for the compensation of the nonlinear dynamics of the synchronization errors $\boldsymbol{e}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$. The feedback linearization creates a new input $\tilde{u}_i$ through which the I/O-behavior between the input $\tilde{u}_i$ and the output $\boldsymbol{e}_{ij}$ becomes linear. Third, a dead-beat controller $\boldsymbol{k}^T$ together with a zero-order hold (ZOH) is designed for the synchronization of the coupled pair of agents in finite time.

Asymptotic synchronization (1) of the overall systems is achieved when the CU establishes couplings of changing pairs of agents in a way that the agents are uniformly connected:

Communication unit
The communication unit has to organize the information exchange between the agents, which is generally done in the following three steps:

1. Find a neighbor j of agent i so that $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| > \gamma$.
2. Establish the connection to the j-th neighbor.
3. Disconnect the agents and go to step 1 if $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| = 0$.

4 Project aims

Synchronization of multi-agent systems have been investigated in [1–4], with respect to the transient behavior. It was shown that by using LQ-control methods optimal synchronization of completely networked multi-agent systems can be achieved.

However, the present work addresses the synchronization problem for a continuous information exchange and linear systems. In this project it is assumed that a coupling among the agents can only be introduced by a pairwise information exchange. Therefore, the goal of this project is to develop design methods for networked controllers with the following properties:

- The networked controller should be robust against changes in the network structure.
- The design of the local algorithms should be independent of the network topology.
- The plug-in of new agents to the network should work without a redesign of the local control algorithms R_i.

5 Example: Rössler oscillators

Synchronization of $N = 10$ identical Rössler oscillators is considered as an application example. The Rössler oscillator is for example useful in modeling chemical reactions or electrical oscillators, which are used in integrated circuits. In particular, the Rössler oscillators display chaotic behavior and are therefore not easy to synchronize.

Figure 3(a) shows the attractor of the Rössler oscillators which have the state space models

$$\dot{\boldsymbol{x}}_i(t) = \begin{pmatrix} -x_{2i}(t) - x_{3i}(t) \\ x_{1i}(t) + 0.2\, x_{2i}(t) \\ 0.2 + x_{3i}(t)\,(x_{1i}(t) - 5.7) + u_i(t) \end{pmatrix}, \ \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i}.$$

The transient behavior of the oscillators is shown in Fig. 4. For randomly chosen inital conditions the simulation

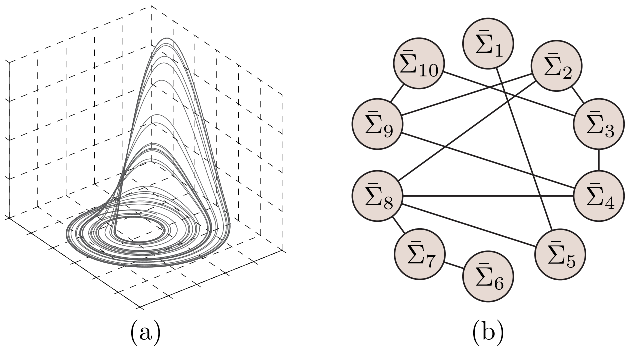

Figure 3: (a) Rössler attractor (b) Communication network $\boldsymbol{L}$

reveals a synchronization of the overall system, which is done by a pairwise coupling of the state signals $\boldsymbol{x}_i$. After $t = 70$ seconds the agents are synchronized.

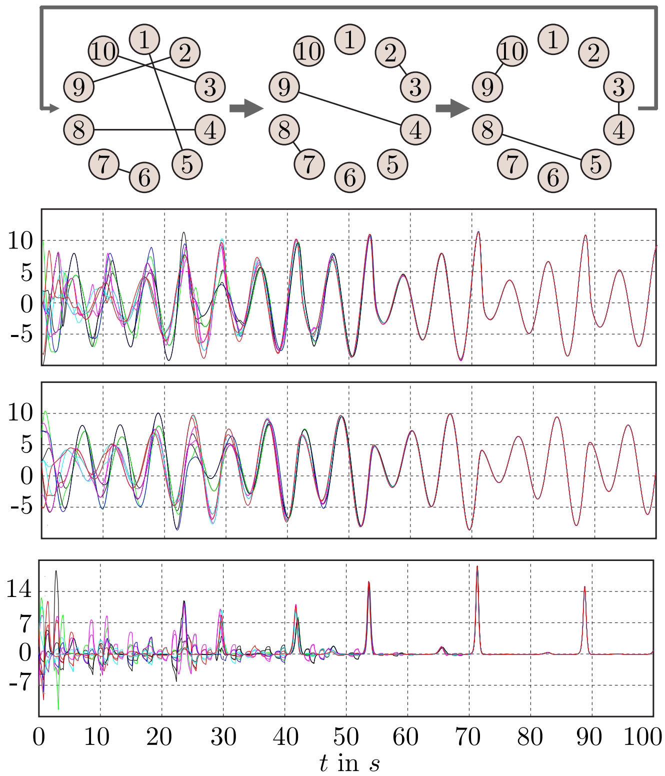

Figure 4: Synchronization of $N = 10$ Rössler attractors

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