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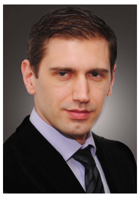

Synchronization of autonomous agents through time-limited output feedback

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1 Introduction

Uncoupled subsystems (often referred to as agents) can be synchronized by a networked controller only if couplings among the agents are introduced in which the input of every agent is influenced by every other agent directly or through a path of agents. Figure 1 illustrates the coupling of a multi-agent system networked by local control algorithms R_i, which are able to exchange information over the communication network $\boldsymbol{L}$.

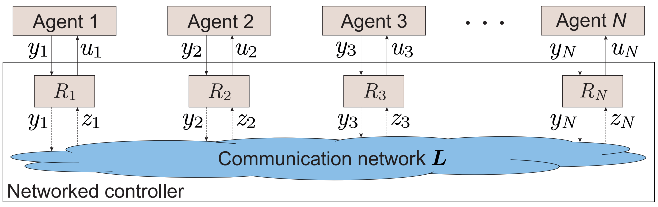

Figure 1: Networked multi-agent system

Typically, synchronization is considered for static or randomly changing network structures. Besides this types of networks there also exists wireless ad hoc networks, where information can only be exchanged by point to point connections.

Hence, new control methods have to be developed for the synchronization of autonomous agents by a pairwise information exchange. A focus of this project is to reduce the communication load in the network, by using a coupling between agents only if there is a sufficiently large deviation between the agent states.

Therefore, this project investigates practical synchronization

$$\lim_{k\to\infty} \|\boldsymbol{y}_i(k) - \boldsymbol{y}_{\mathrm{s}}(k)\| \leq \epsilon, \qquad i, j = 1, 2, \ldots, N.$$

Applications to such kind of synchronization problems are for example sensor networks, which are used for field monitoring. The aim is to improve the determination of physical parameters as temperature or pressure by averaging/synchronizing the data from a large number of locally distributed sensors.

This project investigates the synchronization of a multi-agent system, where couplings between agents are only introduced if the synchronization condition (1) is violated. Based on the restriction of a point to point communication network, new design methods have to be investigated for the synchronization of the agents.

2 System model and network

The agents are described by the time-discrete models

$$\Sigma_i : \begin{cases} \boldsymbol{x}_i(k+1) = \boldsymbol{A}\,\boldsymbol{x}_i(k) + \boldsymbol{b}\,u_i(k), & \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i} \\ y_i(k) = \boldsymbol{c}^T\boldsymbol{x}_i(k), \end{cases}$$

where $\boldsymbol{x}_i$ is the state vector, u_i the scalar control input and y_i the output signal of the i-th agent.

The communication network is described by a bidirectional graph and the associated Laplacian matrix $\boldsymbol{L}$, where the i-th and j-th agents are referred to as neighbors if $(\boldsymbol{L})_{ij} = 1$. In this project it is assumed that the communication network $\boldsymbol{L}$ has the following properties:

- Each agent can only be connected to one of its neighbors at the same time.
- Disjoint pairs of agents can be connected at the same time.
- The communication network is instantaneous and lossless.

3 Control algorithm

Figure 1 illustrates the structure of a networked controller, where local control algorithms R_i together with the communication network $\boldsymbol{L}$ are used to introduce a coupling among the agents.

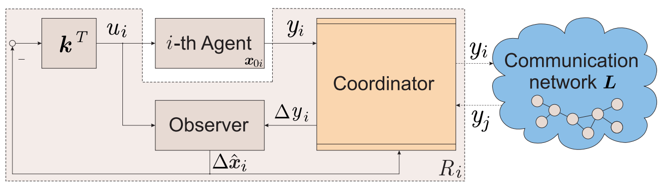

Figure 2: Structure of the control algorithm

The local control algorithms have the structure shown in Fig. 2. The idea behind the structure of the algorithm is to create a coupling between two agents for a period of time, which is required to synchronize the pair of agents. After connecting the agents, the synchronization is obtained by an observer-based state-feedback.

Coordinator
The coordinator has to organize the information exchange between the agents, which is done in the following three basic steps:

1. Find a neighbor j of agent i so that $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| > \gamma$.
2. Establish the connection to the j-th neighbor.
3. Disconnect the agents and go to step 1 if $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| = 0$.

Observer and Controller
The observer should estimate the state difference between two coupled agents in $k = n$ steps. The state-feedback should synchronize the agents after additional $k = n$ steps, so that two coupled agents are completely synchronized after $k = 2n$ steps.

4 Project aims

Synchronization of multi-agent systems has been investigated in [1–4], with respect to the transient behavior. It was shown that by using LQ-control methods optimal synchronization of completely networked multi-agent systems can be achieved.

However, the present work addresses the synchronization problem for a continuous information exchange, where the states are communicated. In this project a coupling among the agents can only be introduced by a pairwise information exchange. In real applications such kind of agent networks appears in systems where the communication topology as well as the number of agents changes.

Therefore, the goal of this project is to develop design methods for networked controllers with the following properties:

- If the synchronization condition (1) is fulfilled, the coupling between agents should no longer need.
- The networked controller should be robust against changes in the network structure.
- The design of the local algorithms should be independent of the network topology.
- The plug-in of new agents to the network should be possible without a redesign of the local control algorithms R_i.

5 Example: harmonic oscillators

Synchronization of identical harmonic oscillators is considered as an application example. There is a variety of real systems which can be modeled by harmonic oscillators. Examples are chemical reactions, in which the state of a substance changes periodically or even electrical oscillators, which are used for all kind of integrated circuits.

Figure 3 shows the synchronization of a network of 15 coupled oscillators with time-discrete state space models

$$\boldsymbol{x}_i(k+1) = \begin{pmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{pmatrix} \boldsymbol{x}_i + \begin{pmatrix} \frac{1-\cos(\omega T)}{\omega} \\ \frac{\sin(\omega T)}{\omega} \end{pmatrix} u_i,$$

$$y_i = \begin{pmatrix} 1 & 0 \end{pmatrix} \boldsymbol{x}_i, \quad \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i}.$$

The transient behavior of the oscillators is shown in Fig. 3. For randomly chosen inital conditions the simulation reveals a synchronization of the overall system, which is done by a parwise coupling of the output signals y_i. After $k = 120$ steps no communication is needed and the agents are synchronized within the bound (1).

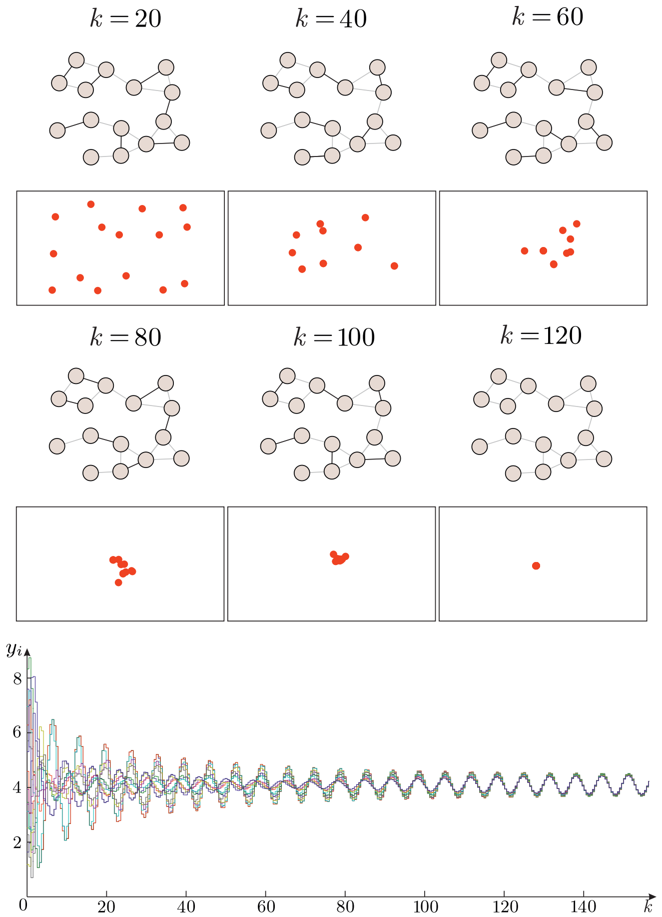

Figure 3: Synchronization of $N = 15$ harmonic oscillators

References

[1] Lunze J. Synchronizable nodes in networked systems. *J. Phys. A: Math. & Theor.*, **44** 045103, 2011

[2] Mosebach A. and Lunze J. Optimal synchronization of circulant networked multi-agent systems. *Proc. of 12th Europ. Contr. Conf.*, pp. 3815–3820, 2013

[3] Mosebach A. and Lunze J. LQR design of synchronizing controllers for networked multi-agent systems. *at-Automatisierungstechnik*, 2014 (submitted)

[4] Mosebach A. and Lunze J. Synchronization of autonomous agents by an optimal networked controller. *Proc. of 13th Europ. Contr. Conf.* , 2014 (accepted)