Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 24096
Fax +49 - (0)234 32 - 14101

Synchronization of multi-agent systems with similar dynamics

Andrej Mosebach
mosebach@atp.rub.de

1 Introduction

Uncoupled subsystems (often referred to as agents) can be synchronized by a networked controller only if couplings among the agents are introduced in which the input of every agent is influenced by every other agent directly or through a path of agents. Therefore, local controllers are used which are coupled by relative output measurements to steer the agents towards a common trajectory $\boldsymbol{y}_{\mathrm{s}}(t)$

$$\lim_{t\to\infty} \|\boldsymbol{y}_i(t) - \boldsymbol{y}_{\mathrm{s}}(t)\| = 0, \qquad i, j = 1, 2, \ldots, N. \qquad (1)$$

Figure 1 illustrates the coupling of a multi-agent system by a networked controller over a communication network $\boldsymbol{L}$, which in this project is assumed to be static, instantaneous and lossless.

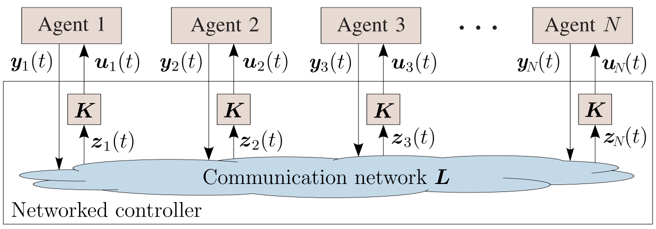

Figure 1: Networked multi-agent system

Applications to the synchronization problem of agents with individual dynamics (also said to be heterogeneous) can be found in various areas such as formation control, communication systems or flocking. Since a lot of physical systems are not individual at all, this project concerns the synchronization problem for a special class of individual systems, the so-called systems with similar dynamics [5]. The agents are assumed to be similar in the sense that a part of the dynamics is common and the remaining part is individual. Considering the example of vehicle platooning, it is obvious that even if the vehicles have different engines or load, synchronization of the vehicles distance can be achieved (Fig. 3).

This project investigates the synchronization of individual agents. Based on the common dynamics of individual agents, control methods have to be investigated for the design of synchronizing networked controllers.

2 Agents with similar dynamics

The state space model of N agents with individual dynamics is given by

$$\Sigma_i : \begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i \boldsymbol{x}_i(t) + \boldsymbol{B}_i \boldsymbol{u}_i(t), & \boldsymbol{x}_i(0) = \boldsymbol{x}_{i0} \\ \boldsymbol{y}_i(t) = \boldsymbol{C}_i \boldsymbol{x}_i(t), & \end{cases}$$

where $\boldsymbol{x}_i(t) \in \mathbb{R}^{n_i}$ is the state, $\boldsymbol{u}_i(t) \in \mathbb{R}^{m_i}$ the control input and $\boldsymbol{y}_i(t) \in \mathbb{R}^{r_i}$ the output signal of the i-th agent. In order for the agents to be similar, it is necessary that the dimensions of the state-space models Σ_i are identical:

$$n_i = n, \quad m_i = m, \quad r_i = r, \quad i = 1, 2, \ldots, N.$$

As an example, consider vehicle platooning, where each of the vehicles have a different mass, velocity-dependent resistance or friction. Note that all of the vehicles, have the same order of input-, output- and state-signals.

The sufficient condition on the agent dynamics to be similar is the existence of regular transformation matrices $\boldsymbol{T}_i$ such that

$$\boldsymbol{T}_i^{-1} \boldsymbol{A}_i \boldsymbol{T}_i = \begin{pmatrix} \boldsymbol{A}_{\mathrm{s}} & \boldsymbol{O} \\ \boldsymbol{O} & \tilde{\boldsymbol{A}}_i \end{pmatrix}, \quad \boldsymbol{T}_i^{-1} \boldsymbol{B}_i = \begin{pmatrix} \boldsymbol{B}_{\mathrm{s}} \\ \tilde{\boldsymbol{B}}_i \end{pmatrix},$$

$$\boldsymbol{C}_i \boldsymbol{T}_i = \begin{pmatrix} \boldsymbol{C}_{\mathrm{s}} & \tilde{\boldsymbol{C}}_i \end{pmatrix}, \quad i = 1, 2, \ldots, N$$

hold. $\boldsymbol{A}_{\mathrm{s}} \in \mathbb{R}^{p\times p}$, $\boldsymbol{C}_{\mathrm{s}} \in \mathbb{R}^{r\times p}$, $\boldsymbol{B}_{\mathrm{s}} \in \mathbb{R}^{p\times m}$, and $\tilde{\boldsymbol{A}}_i$, $\tilde{\boldsymbol{B}}_i$, $\tilde{\boldsymbol{C}}_i$ are matrices of appropriate dimensions.

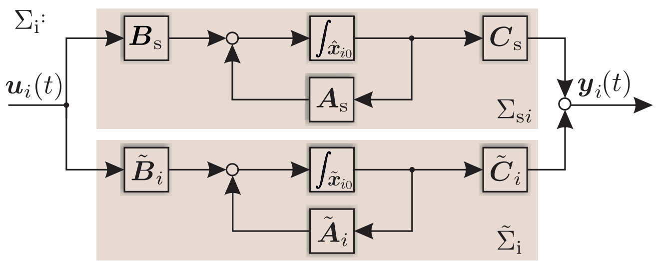

Figure 2: Structure of similar systems

The structure of the state-space models Σ_i plays an important role for the design of synchronizing networked controllers [1]. As shown in Fig. 2 all state-space models of agents with similar dynamics can be decomposed into common dynamics $\Sigma_{\mathrm{s}i}$ and into individual dynamics $\tilde{\Sigma}_i$. From the initial conditions $\boldsymbol{x}_{\mathrm{s}i0} = \boldsymbol{x}_{\mathrm{s}0}$ and $\boldsymbol{x}_{i0} = 0$, $(i = 1, 2, \ldots, N)$, it can be seen that the agents can move synchronously $\boldsymbol{y}_i(t) = \boldsymbol{y}_{\mathrm{s}}(t)$.

April 24, 2013

3 Networked controller

Figure 1 illustrates the structure of a networked controller, where local controllers $\boldsymbol{K}$ together with the communication network $\boldsymbol{L} = (l_{ij})$ are used to introduce a coupling among the agents. $\boldsymbol{L}$ is well known as the Laplacian matrix of a directed graph, which is used to represent the topology of the communication network (e.g. Fig. 4). The coupling signals

$$\boldsymbol{z}_i(t) = \sum_{j=1}^{N} l_{ij} \left(\boldsymbol{y}_i(t) - \boldsymbol{y}_j(t)\right)$$

are a linear combination of the i-th agents output relative to that of neighboring agents, where the i-th and the j-th agents are said to be neighbors if $l_{ij} \neq 0$.
The local control laws should be designed so that the coupling signals $\boldsymbol{z}_i(t)$ are used to manipulate the agents behavior by identical controller matrices $\boldsymbol{K}$:

$$\boldsymbol{u}_i(t) = \boldsymbol{K}\boldsymbol{z}_i(t).$$

4 Project aims

Synchronization of identical agents have been investigated in [2–4], where conditions on synchronization and control methods with respect to the transient behavior are derived. It was shown that by using LQ-control methods an optimal synchronization of completely networked multi-agent systems can be achieved.
However, the present work only addresses the synchronization problem for identical agents and bidirectional communication networks, where the states are communicated.
The aim of this project is to develop design methods for networked controllers to synchronize multi-agent systems under the following assumptions:

- The agent dynamics are assumed to be similar.
- Each agent has only knowledge about its own coupling signal $\boldsymbol{z}_i(t)$.
- The communication network $\boldsymbol{L}$ is assumed to be directed.

As a major goal of this project the design method should, in addition to the requirement (1), be elaborated such that one of the following performance indices is satisfied:

- Requirement on the synchronization within the time bound T_{sync}:
$$\lim_{t\to\infty} \|\boldsymbol{y}_i(t) - \boldsymbol{y}_{\text{s}}(t)\| < \epsilon, \quad t \geq T_{\text{sync}}.$$
- Requirement of the minimization of LQ-like objective functions, as
$$J = \int_0^\infty \sum_{i,j} \left(\boldsymbol{y}_i(t) - \boldsymbol{y}_j(t)\right)^T \boldsymbol{Q}_{ij} \left(\boldsymbol{y}_i(t) - \boldsymbol{y}_j(t)\right) \mathrm{d}t.$$

The complexity of designing networked controllers grows with the number of agents. Hence, decomposition methods have to be investigated, in order to reduce the computational load.

5 Example: Vehicle platoon

Distance control of non-identical vehicles is considered as an application example (Fig. 3). Each of the vehicles is assumed to be equipped with a front distance sensor and a wireless communication module, which allows to transmit data among vehicles. If there is no inter-vehicles communication by using the wireless modules, then the topology of the networked controller reduces to the chain shown in Fig. 4a. It is possible to use the wireless communication in order to improve the synchronization behavior of the vehicle platoon. The more communication links are used the faster the vehicles can be influenced by deviations on the desired distance. In the case, where the initial position of only the first vehicle is picked to be different from zero Fig. 4b shows a network topology which clearly improves the behavior of the vehicle platoon.

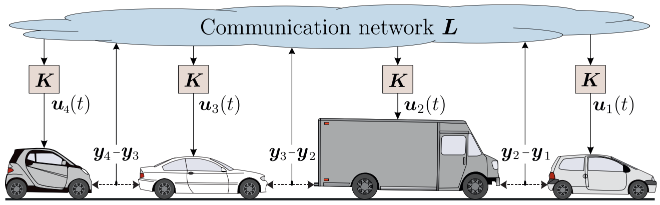

Figure 3: Vehicle platoon

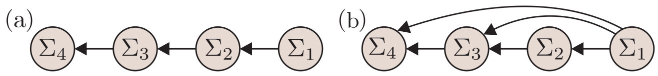

Figure 4: Communication topology

It is clear that the transient behavior of the networked vehicles relates directly to the behavior of the slowest vehicle in the platoon. To develop controller design methods which take the different dynamics of the vehicles into account is one of the main goals.
However, as there is a variety of communication topologies, new design methods for networked controllers have to be investigated. The focus of the design methods should be the transient behavior with respect to the communication topology.

References

[1] Lunze J. Synchronization of heterogeneous agents. *IEEE Trans. on Autom. Contr.*, 2012

[2] Lunze J. Synchronizable nodes in networked systems. *J. Phys. A: Math. & Theor.*, 2011

[3] Mosebach A. and Lunze J. LQR design of synchronizing controllers for networked multi-agent systems. *Proc. of 52th Conf. on Decis. & Contr.*, 2013 (submitted)

[4] Mosebach A. and Lunze J. Optimal synchronization of circulant networked multi-agent systems. *Proc. of 12th Europ. Contr. Conf.* , 2013 (accepted)

[5] Mosebach A. and Lunze J. Synchronization of multi-agent systems with similar dynamics. *4th IFAC Worksh. Distrib. Estim. & Contr. in Netw. Sys.*, 2013 (submitted)