Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150 | Phone +49 - (0)234 32 - 24096
D-44805 Bochum, Germany | Fax +49 - (0)234 32 - 14101

Synchronization of Multi-Agent Systems

Andrej Mosebach
mosebach@atp.rub.de

1 Introduction

Dynamical systems consisting of a large number of decoupled identical subsystems are called multi-agent systems. Each subsystem is equipped with actuating and sensing units.
In this project networked multi-agent systems consisting of physically unconnected subsystems are considered (Figs. 1 and 2). Using controllers the subsystems can be linked to an arbitrary communication network.

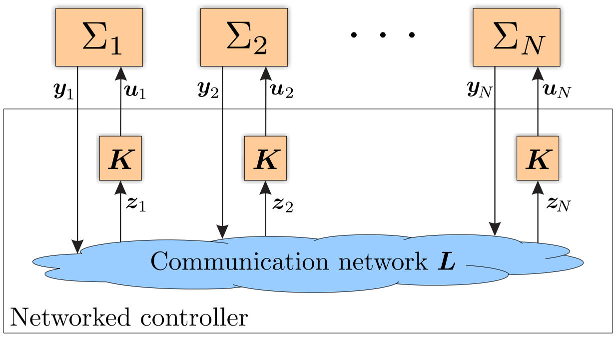

Figure 1: Multi-agent system linked to a uniformly weighted communication topology by networked controller

The dynamical behavior of networked systems strongly depends on the network structure (communication topology) and the coupling strength (controller). One important phenomenon which depends upon the controller parameter is that even if the networked multi-agent system is unstable, the output difference between all subsystems vanishes asymptotically for arbitrary initial conditions:

$$\lim_{t\to\infty} \left\| \boldsymbol{y}_i(t) - \boldsymbol{y}_j(t) \right\| = 0, \quad i = 1, 2, \ldots, N.$$

This is referred to as asymptotic synchronization [1].
The project investigates methods for the design problem of synchronizing networked controllers for multi-agent systems with identical linear dynamics.

2 Communication networks

Depending on the structure of the communication network it is possible to solve the synchronous controller design problem using linear quadratic regulator (LQR) methods. In the following two possible network structures are presented.

2.1 Uniformly weighted completely connected networks

In completely connected networks each subsystem is connected to every other subsystem in uniform manner:

$$\boldsymbol{z}_i(t) = \sum_{j=1}^{N} \left(\boldsymbol{y}_i(t) - \boldsymbol{y}_j(t) \right).$$

If the full state vector is measurable it is possible to formulate the objective function in terms of the uniformly weighting coupling signal $\boldsymbol{z}_i(t)$ to find an admissible controller. Minimizing this quadratic objective function subjected to the multi-agent system yields to a control law, which provides the uniformly weighted completely connected communication topology (Fig. 1).
Typically, multi-agent systems consist of a large number of subsystems. With the increase of subsystems the complexity of the minimization problem rises. Using a decomposition of the objective function the computational effort of the LQR design problem can be reduced to the subsystem order.
In this type of completely connected networks the coupling strength among all subsystem pairs is identical. Hence, if a single subsystem is disturbed, the response of all other subsystems will be identical. However, in many application examples the multi-agent system consists of groups of subsystems which have higher priority to share information then others. In control of adaptive optics, for example, it is of great interest that directly neighboring subsystems are stronger coupled then the other subsystems. This kind of behavior cannot be achieved using uniformly weighted completely connected networks as explained in this section.

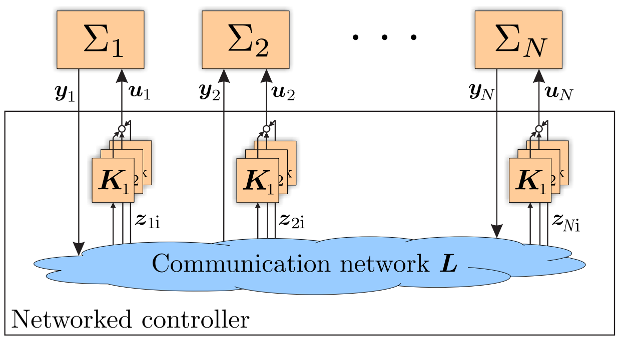

Figure 2: Multi-agent system linked to an arbitrary weighted communication topology by networked controller

April 16, 2012

2.2 Circulant weighted completely connected networks

Considering that all subsystems are arranged in a ring the circulant weighting means that directly located subsystems (one-step neighbors) are coupled with the coupling strength $\boldsymbol{K}_1$, two-step neighbors with $\boldsymbol{K}_2$ and the next steps similarly (Fig. 3).

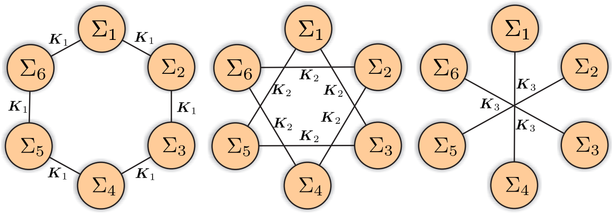

Figure 3: Circulant weighted network

Defining the LQR problem in terms of circulant weighting matrices yields to a networked controller with same structure (Fig. 2). The computational effort for the solution of the LQR problem subjected to the overall system can be reduced to the solution of $N/2$ algebraic Riccati equations subjected to one subsystems order.
The synchronization behavior of such networked systems is much different to the case of uniformly weighting networks. Subsystems which are coupled through a strong coupling strength synchronize, even before all subsystems share a common trajectory.
The vehicle merging control problem of automated highway systems provides an application example for the mentioned network structure. Since the behavior of the first vehicle should not depend on the behavior of the last vehicle as much as on the directly located vehicles.

3 Project aims

The main focus of the project is to design networked synchronizing controllers for linear multi-agent systems with identical subsystem dynamics. Since the networked multi-agent system does not have to be stable to reach synchronous behavior of the subsystems [1], the first step is to consider the dynamics of the synchronization error

$$\boldsymbol{e}_{ij}(t) = \boldsymbol{y}_i(t) - \boldsymbol{y}_j(t).$$

It was already mentioned that using LQR methods, with objective functions containing terms of the synchronization error, it is possible to find synchronizing state feedback controllers. This networked controller matrices are typically completely filled and, therefore, restricted to completely connected communication networks.
Considering completely connecting state feedback controllers the extensions to the design of synchronizing output-feedback controller for communication networks with a incompletely connecting structure have to be made. Since the complexity of designing networked controllers for multi-agent systems grows with the number of subsystems more than linearly a decomposition of the synchronizing controller design problem have to be investigated.

In the view of the merging control example it is necessary that not only the synchronous state is reached asymptotically but also that the vehicles are approximately synchronized at the merging point. Therefore, the designed controller have to ensure a maximum quality of transition behavior.

4 Application example

The aims of automated highway systems are both, to improve the traffic flow and to avoid vehicle accidents. On road constrictions the vehicles have to meet a given distance to pass the merging point without a collision (Fig. 4). This task can be solved using a synchronization of the vehicle platoons in terms of the vehicle distance and velocity.

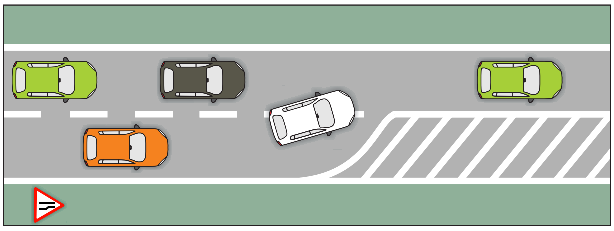

Figure 4: Synchronization of vehicles on merging points

Before the first vehicle passes the merging point, the vehicles on both highway lanes must achieve the synchronous behavior. Independent on the initial speed and the type of additional incoming vehicles the networked controller have to ensure the synchronization of the merging point leaving vehicles.
This example states many problems which have to be solved. With the use of networked controllers which only ensure asymptotic synchronization it cannot be guaranteed that the vehicles are synchronized after a maximum time interval. This dynamical behavior have explicitly to be considered in the controller design method. Since the dynamical behavior of individual vehicles depends on various parameters, new challenges in the design of synchronizing controllers arise. Controller design methods for multi-agent systems with individual dynamics are of great interests [2]. The changing number of vehicles leads to another problems a time variant communication topology and a time variant dimension of the multi-agent system.

References

[1] Lunze, J.: Synchronisation autonomer Agenten durch vernetzte Regelung, *Ruhr-Universität, Lehrstuhl für Automatisierungstechnik und Prozessinformatik,* 2010

[2] Lunze, J.: An internal-model principle for the synchronisation of autonomous agents with individual dynamics, *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on,* 2011, 2106-2111