Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 28071
Fax +49 - (0)234 32 - 14101

# Consistency-based diagnosis using set observation

**Dipl.-Ing. Philippe Planchon**
**planchon@atp.ruhr-uni-bochum.de**

## 1 Model-based diagnosis

The aim of diagnosis is to determine if a running process is affected by some fault $f^\circ$. The task is pursued using input and output measurements, which represent the current behaviour of the process. The general structure of a diagnosis scheme is shown in Fig. 1.

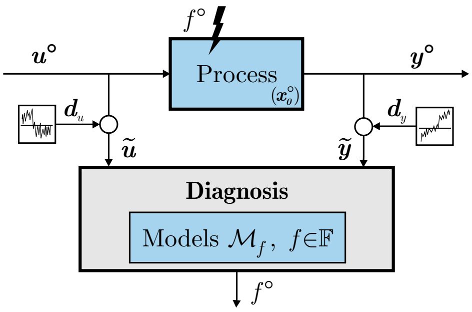


Figure 1: Ideal diagnosis

State of the art diagnosis implements additionally one or more models which characterise the process dynamics for specific fault cases, [1]. As such, model-based diagnosis must determine the match between input-output (I/O) behaviour and the given models. In this project, the emphasis lies on the consideration of process uncertainty which is caused either by erroneous measurements or by approximate models. A method is sought which offers robustness with respect to these uncertainties.

## 2 Framework of operation

Let $\mathbb{F} = \{f_0, f_1, \dots, f_N\}$ be the set containing all possible fault cases under consideration, with $f_0$ describing the faultless process. Each fault $f \in \mathbb{F}$ has a model $\mathcal{M}_f$ – considered here to be precisely known – described by the discrete-time system

$$\boldsymbol{x}(k+1) = \boldsymbol{A}_f\,\boldsymbol{x}(k) + \boldsymbol{B}_f\,\boldsymbol{u}(k), \qquad (1)$$
$$\boldsymbol{y}(k) = \boldsymbol{C}_f\,\boldsymbol{x}(k) + \boldsymbol{D}_f\,\boldsymbol{u}(k). \qquad (2)$$

The true input $\boldsymbol{u}^\circ$ and output $\boldsymbol{y}^\circ$ are subject to measurement errors $\boldsymbol{d}_u$ and $\boldsymbol{d}_y$ and, hence, are unknown. As opposed to many methods which consider probabilistic distribution for these errors, an unknown-but-bounded assumption is favoured in the following. This suits practical applications for which stochastic information is unknown or which inappropriately describe the actual error (*e.g.* sensor offsets). Therefore, two bounds

$$|\boldsymbol{d}_u(k)| \le \boldsymbol{e}_u(k) \quad \text{and} \quad |\boldsymbol{d}_y(k)| \le \boldsymbol{e}_y(k) \qquad (3)$$

are supposed known and, using the measured input $\tilde{\boldsymbol{u}}$ and output $\tilde{\boldsymbol{y}}$, two sets

$$\mathcal{U}(k) = \{\boldsymbol{u} \in \mathbb{R}^m \mid |\tilde{\boldsymbol{u}}(k) - \boldsymbol{u}| \le \boldsymbol{e}_u(k)\} \qquad (4)$$
$$\text{and} \quad \mathcal{Y}(k) = \{\boldsymbol{y} \in \mathbb{R}^r \mid |\tilde{\boldsymbol{y}}(k) - \boldsymbol{y}| \le \boldsymbol{e}_y(k)\} \qquad (5)$$

are described which guaranteed to contain the true input and output

$$\boldsymbol{u}^\circ(k) \in \mathcal{U}(k) \quad \text{and} \quad \boldsymbol{y}^\circ(k) \in \mathcal{Y}(k). \qquad (6)$$

## 3 Robust diagnostic approach

Complex faults may only be distinguished from the faultless operation by analysing the dynamic behaviour. For this reason, sequences of I/O over a finite time horizon $[0, \bar{k}]$ are considered

$$\boldsymbol{U} = \boldsymbol{U}(0\dots\bar{k}) = \big(\boldsymbol{u}(0), \boldsymbol{u}(1), \dots, \boldsymbol{u}(\bar{k})\big),$$
$$\boldsymbol{Y} = \boldsymbol{Y}(0\dots\bar{k}) = \big(\boldsymbol{y}(0), \boldsymbol{y}(1), \dots, \boldsymbol{y}(\bar{k})\big).$$

Based on the unknown-but-bounded assumption (3), the diagnosis must determine which of the dynamic models $\mathcal{M}_f$, $f \in \mathbb{F}$, may generate an I/O within the sequences of sets

$$\mathcal{U} = \mathcal{U}(0\dots\bar{k}) = \big(\mathcal{U}(0), \mathcal{U}(1), \dots, \mathcal{U}(\bar{k})\big), \qquad (7)$$
$$\mathcal{Y} = \mathcal{Y}(0\dots\bar{k}) = \big(\mathcal{Y}(0), \mathcal{Y}(1), \dots, \mathcal{Y}(\bar{k})\big). \qquad (8)$$

Such a model is said to be consistent with the sequence of I/O sets and is noted

$$\mathcal{M}_f \models (\mathcal{U}, \mathcal{Y})(0\dots\bar{k}). \qquad (9)$$

As the consistent model may not be unique, a robust diagnosis is aimed to describe the set of fault candidates

$$\mathcal{F}^*(k) = \{f \in \mathbb{F} \mid \mathcal{M}_f \models (\mathcal{U}, \mathcal{Y})(0\dots k)\}.$$

The method is described as *consistency-based* and its result is then guaranteed to always include the true fault affecting the process: $f^\circ \in \mathcal{F}^*(k)$, $0 \le k \le \bar{k}$.

The consistency (9) may be tested using a set-membership state observation algorithm, as described in [2, 3]. Such an algorithm describes the set $\mathcal{X}_f(\bar{k})$ of states $\boldsymbol{x}(\bar{k})$ which may be reached under consideration of the model $\mathcal{M}_f$ and the sequence of I/O sets (7)–(8). If any such state exists, *i.e.* $\mathcal{X}_f(\bar{k}) \neq \varnothing$, then the corresponding fault may have occurred. Derived from this principle, a recursive diagnostic algorithm is constructed as follows (and illustrated in Fig. 2).

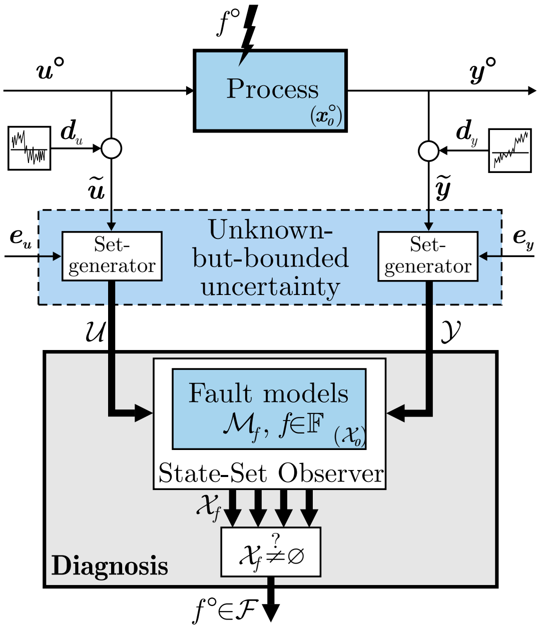


Figure 2: Consistency-based diagnosis using set observers

**Consistency-Based Diagnostic Algorithm**

GIVEN:

- The sequence of I/O sets $(\mathcal{U}, \mathcal{Y})(0 \ldots \bar{k})$
- The linear models $\mathcal{M}_f, \forall f \in \mathbb{F}$

LOOP: (Initialise $k := 0$ and $\mathcal{F}(-1) := \mathbb{F}$)

1. $\forall f \in \mathcal{F}(k-1)$, compute $\mathcal{X}_f(k)$ the state-set observation for model $\mathcal{M}_f$ and I/O sets $(\mathcal{U}, \mathcal{Y})(0 \ldots k)$, as in [2].
2. Preserve fault models which are not inconsistent:
$$\mathcal{F}(k) := \{f \in \mathcal{F}(k-1) \mid \mathcal{X}_f(k) \neq \varnothing\}.$$
3. If $k < \bar{k}$, then $k := k+1$ and go to Step 1.

RESULT:

- Set of fault models $\mathcal{F}(\bar{k})$.

Due to simplifications implemented to render the set-calculations computationally feasible, the diagnostic algorithm only computes an over approximation of the set of fault candidates. This relation, however, preserves the guaranteed diagnostic result:

$$f^\circ \in \mathcal{F}^*(\bar{k}) \subseteq \mathcal{F}(\bar{k}).$$

# 4 Example of robust diagnosis

The method is illustrated using a second-order process. Three behaviours $\mathbb{F} = \{f_0, f_1, f_2\}$ are considered for the diagnosis. The I/O are measured while the process is subject to fault $f^\circ := f_2$ and are erroneous, but contained in the bounds described by

$$\boldsymbol{e}_u(k) = 0.08 \qquad \text{and} \qquad \boldsymbol{e}_y(k) = 0.10\,|\tilde{\boldsymbol{y}}(k)|.$$

Fig. 3 depicts the results of the three state-set observations $\mathcal{X}_f(k)$, $f \in \mathbb{F}$. The consistency test results in the following sets:

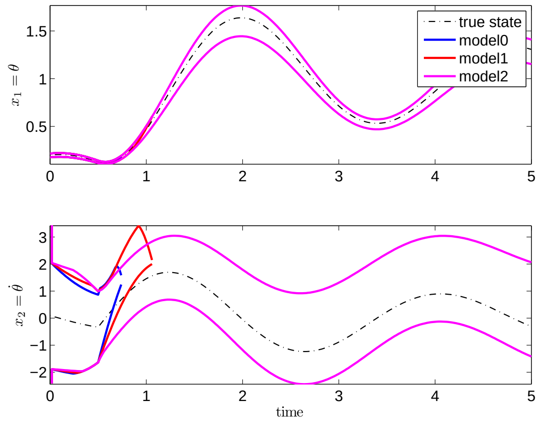


Figure 3: State-set observation results

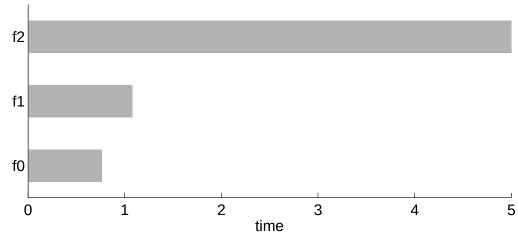


Figure 4: Consistency result

$$\mathcal{F}(t) = \begin{cases} \{f_0, f_1, f_2\} & \text{if } 0 \leq t < 0.74\,\text{s} \\ \{f_1, f_2\} & \text{if } 0.74\,\text{s} \leq t < 1.06\,\text{s} \\ \{f_2\} & \text{if } 1.06\,\text{s} \leq t \end{cases}$$

as seen in Fig. 4 (expressed in continuous-time).
A further example, obtained for an industrial application, is found in [4, 5].

Current research interest lies in the extension of the methodology to consider modelling uncertainties, as well as the analysis of diagnosibility conditions. These topics are closely related to similar issues regarding the state-set observer.

# References

[1] Blanke, M.; Kinnaert, M.; Lunze, J.; Staroswiecki, M.: *Diagnosis and Fault-Tolerant Control*, Springer-Verlag, Heidelberg, 2003.

[2] Planchon, P.: *Polytopic state-set observation: Completeness and application to diagnosis*, Ruhr-Universität Bochum, Lehrstuhl für Automatisierungstechnik und Prozessinformatik Bochum, 2006.

[3] Planchon, P.; Lunze, L.: *Robust diagnosis using state-set observation*, Proceedings of SAFEPROCESS, Beijing, China, 2006 (to appear).

[4] Planchon, P.; Lunze, L.; Rode, M.: *Diagnosis of the unwinding process of a rolling mill* ATPi, April 2006 (to appear).

[5] Lunze, J.; Planchon, P.: *Development and Testing of Process Supervision Methods using Semi-Quantitative Models for Cold Rolling Mills*, ABB DECRC Ladenburg, September 2004.