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Analysis and Control of Discretely Controlled Continuous Systems

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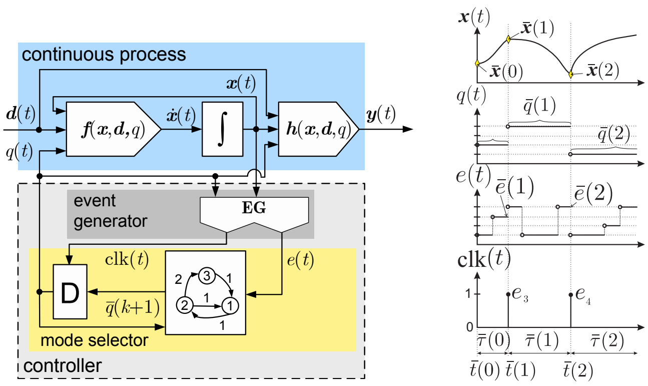
1 Discretely controlled continuous systems

Discretely controlled continuous systems (DCCS) constitute a special subclass of hybrid systems, which is of great practical relevance. Such systems are found in many application domains such as production engineering, power electronics and process engineering.



(a) block diagram (b) trajectories

Figure 1: Structure of a discretely controlled continuous system

As depicted in Figure 1, DCCS are composed of a continuous process and a discrete-event controller arranged in a feedback loop. To meet given specifications, the controller switches among the *modes of operation* $q \in Q$ of the plant, which are associated with distinct continuous dynamics. Important properties of DCCS are:

- The plant has no exogenous continuous inputs.
- In each mode of operation, the evolution of the continuous state is governed by structurally different *mode dynamics*.
- The admissible switching frequency is upper bounded.
- The *activation duration* of each mode is finite, enforcing a chaotic or a periodic stationary behavior.

The latter results from the primary design objective, which is to maintain the state $\boldsymbol{x}(t)$ inside a *terminal region* containing no equilibria. To explicitly account for the distinct control influences

- *successor mode* $\bar{q}(k+1)$ and
- *switching instant* $\bar{t}(k+1)$

on the evolution $\boldsymbol{x}(t)$, the controller is subdivided into

- an *event generator* defining switching conditions and
- a deterministic *switching logic* or *mode selector*, which fixes all feasible mode transitions.

The event generator's task is to select a potential event symbol $e(t)$ by evaluating the signals $\boldsymbol{x}(t)$, $q(t)$ and t, which denote the plant's state, the currently activated mode and time, respectively. The actual occurrence of an event $\bar{e}(k)$ is finally indicated by the impulsive trigger signal $\mathrm{clk}(t)$, which drives the latch D. These events initiate transitions in the switching logic and thus cause mode switches (*controlled switching*).
In addition to the controller dynamics, the discrete-event subsystem may contain components, which account for process inherent *autonomous switching*. Likewise, subsidiary continuous control loops may be lumped into the continuous subsystem.

2 Models

A model suited for describing the complex behaviour of discretely controlled continuous systems is given by the equations

Continuous process:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)), \ \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}(t) &= \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)) \end{aligned}$$

Controller - event generator:

$$\Phi(\boldsymbol{x}, t, q, e) = \begin{cases} \Phi_{e_1}(\boldsymbol{x}, t, q), & \text{if } e = e_1 \\ \vdots & \\ \Phi_{e_E}(\boldsymbol{x}, t, q), & \text{if } e = e_E \end{cases}$$

$$e(t) = \arg\min_{e \in \mathcal{E}} |\Phi(\boldsymbol{x}(t), q(t), t, e)|$$

$$\mathrm{clk}(t) = \begin{cases} 0, & \text{if } |\Phi(\boldsymbol{x}(t), t, q(t), e)| > 0, \forall e \in \mathcal{E} \\ 1, & \text{if } \exists e \in \mathcal{E} \mid \Phi(\boldsymbol{x}(t), t, q(t), e) = 0 \end{cases},$$

$$\bar{t}(k+1) = \min_{t > \bar{t}(k)} t \mid \mathrm{clk}(t) = 1$$

Controller - switching logic:

$$\bar{q}(k+1) = g(\bar{q}(k), \bar{e}(k+1)), \ \bar{q}(0) = q_0$$

where $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ denote the continuous state and the measured output, $\boldsymbol{d}(t)$ denotes continuous disturbances, $q(t)$ denotes the mode of operation and e denotes an event from the set of possible events $\mathcal{E}$. All symbols with bars denote signal values at the instants of switching $\bar{t}(k)$ and the counter k enumerates the switching occurrences. This model can be represented as a hybrid automaton, which is summarized in [2].

When dealing with specific analysis and design tasks, it is beneficial to consider abstractions on different levels, that capture the characteristics essential for solving the task.
Sampled data models describe the evolution of the signal trajectories sampled at the instants of switching [4] and are suited to analyze and enforce periodic stationary behaviour.
If the switching frequency is sufficiently high, *average models* provide a purely continuous abstraction, which allow to derive control laws by methods from classical control theory.
Discrete-event abstractions of the hybrid system represent abstractions suited for the design of controllers that execute set-point transitions [5].

3 Project aims

The project is focused on the development of a holistic and systematic controller synthesis methodology for this class of hybrid systems. The control loop has to satisfy following requirements:

- The continuous state must be globally driven into a specified terminal region $\mathcal{X}_\mathrm{T}$.
- The state must be maintained inside this region $\mathcal{X}_\mathrm{T}$ despite the acting disturbances and parameter variations.
- The number as well as the frequency of mode switches is generally related to costs and therefore must be minimized.
- Due to resource limitations, the control law must be explicit and only require minor computational power.

In this research issues of modeling, analysis, controller synthesis and state observation are considered. Novel stability definitions and methods for analysis have to be developed, as existing methods for switched continuous systems require the system to tend to a common equilibrium instead of a limit cycle [3]. Furthermore analytic criteria and algorithms for checking controllability and stabilizability must be elaborated for these systems. Besides the adaptation of existing design approaches for *switched continuous systems*, such as Lyapunov-based or reachability-based design approaches, novel procedures to enforce a desired periodic stationary behavior must be developed. In the design process, the effects of disturbances as well as parameter variations have to be explicitly considered. The outcome of any DCCS design procedure have to be explicit expressions of the *event function* Φ and the *transition function* g. State reconstruction [1] is an important aspect as well, since in many applications only few measurements are available.

4 Experimental examples

In this project, two applications will be considered for the evaluation of developed methods and algorithms. The first practically relevant application comprises switching power converters, such as a DC-DC boost converter shown in Figure 2. This device generates a high set-point DC voltage from a low DC voltage source by a prolonged switching among the three modes of the system. The closed loop behavior has to satisfy

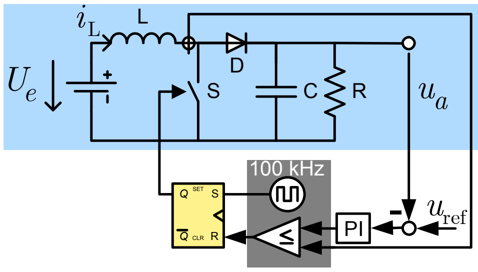


Figure 2: DC-DC boost converter

hard constraints such as a small stationary periodic oscillation of the output voltage u_c around the given setpoint, while showing robustness against large load and input voltage variations.
The second benchmark application is an experimental setup of a flexible manufacturing cell (Figure 3). The objective of the implemented process is to regulate the temperature of a metal block by transferring thermal energy from workpieces, which can be heated or cooled down in other components. The complexity of the control task increases with the number of workpieces and the number of constraints to be satisfied by the workpiece temperatures. In contrast to the boost converter the primary control task comprises the execution of large transitions from one set-point to another, such as in start-up and shut-down procedures of complex plants.

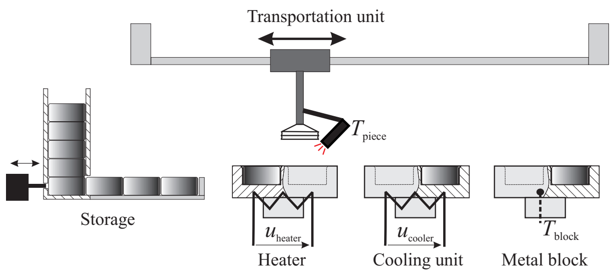


Figure 3: Scheme of the manufacturing process

5 Cooperation

This project is being worked on in cooperation with the Institute for principles of electrical engineering (IEE) at the University of Dresden.

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