Event-based control of input-output linearizable systems ⋆

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Abstract: This paper proposes a new event-based control method for nonlinear systems that are input-output linearizable. The control input generator uses a copy of a continuous reference system to generate an exponential control input that keeps the state of the disturbed plant in a bounded surrounding of the setpoint. An upper bound for the deviation of the event-based control loop to the reference system is derived, which depends on the event threshold of the event generator. Hence, by appropriately choosing the event threshold, the event-based control can be made to mimic the continuous control with arbitrary accuracy. The proposed event-based control method is applied to a cooling process.

Keywords: Event-based control, input-output linearizable system, disturbance rejection, ultimate boundedness, networked control system.

1. INTRODUCTION

1.1 Event-based control

Event-based control is a new control paradigm that aims at reducing the communication between the sensors, the controller and the actuators within a control loop by initiating a communication among these components only after an event has indicated that the control error exceeds a threshold. With this control strategy, the network utilization shall be minimized.

The event-based control loop, as investigated in this paper, is illustrated in Fig. 1. It consists of three parts:

- the plant with input $u(t)$, output $y(t)$, state $\boldsymbol{x}(t)$ and disturbance vector $\boldsymbol{d}(t)$,
- the event generator and
- the control input generator, which incorporates the controller.

The event generator determines the event time instants t_k $(k = 0, 1, ...)$ at which a communication between the event generator and the control input generator is induced and transmits the sensor data, as well as previously processed signals like the disturbance estimation $\hat{\boldsymbol{d}}_k$. The control input generator computes the trajectory of the control input signal $u(t)$ for the time interval $t \in [t_k, t_{k+1})$ in dependence upon the information received at time t_k. In Fig. 1, the dashed arrow indicates that this information link is only used at the event times t_k $(k = 0, 1, ...)$, whereas the solid arrows represent a continuous information transmission.

This paper proposes a design method for the event-based control of nonlinear plants that are input-output linearizable.

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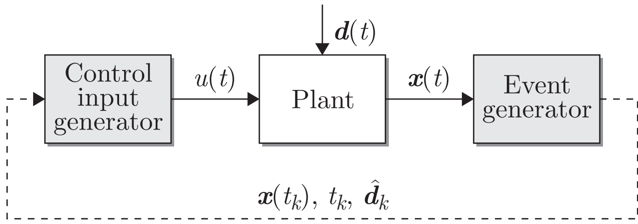

Fig. 1. Event-based control loop

Following the idea of Lunze and Lehmann (2010), the design aim is to make the event-based control loop mimic a continuous state-feedback loop (hereafter referred to as reference system) with prescribed accuracy. Copies of the continuous reference system are used for the control input generation and the event generation. As the linearizing state feedback is applied, the reference system is linear and so are the copies used in both generators. However, due to the disturbance $\boldsymbol{d}(t)$ and the event-based sampling, the generated control input differs from the linearizing input and the main analysis problem to be solved in this paper concerns the question how large the deviation of the event-based version of the feedback and its continuous counterpart is. An upper bound of this deviation will be derived showing that the proposed event-based control method reaches the control aim.

1.2 Literature review

Event-based control is a new research topic, which has been tackled in literature mainly for linear plants, whereas only a few publications deal with nonlinear systems. Wang and Lemmon (2008a) consider the event-based stabilization of an unstable, nonlinear plant. The starting point has been a given Lyapunov function of the continuously controlled system. The idea of this approach is to yield an upper bound of the derivative of the Lyapunov function by an approximation as a function of the plant state. Each time, this approximation tends to take a nonnegative value, the control input is updated in order to stabilize the plant. As a zero-order hold (ZOH) was used as control input generator, the recalculated control input signal is held constant until the next event time instant. This basic idea has been extended in many ways, e. g. to distributed event-based control by Wang and Lemmon (2008b) and event-based control with delays and data dropouts by Wang and Lemmon (2009).

Tabuada (2007) described an event trigger mechanism for the self-triggered stabilization of an unstable plant. In self-triggered control, the next event time is predetermined at the previous event time and does, therefore, not use the current plant information.

This paper extends the work of Lunze and Lehmann (2010) to the event-based disturbance rejection in nonlinear control systems. In contrast to the methods published in literature, the event trigger mechanism depends solely on the measured plant state and no Lyapunov function of the continuous closed-loop system needs to be known. Instead of a ZOH, a smart control input generator is proposed, which generates an exponential control input signal.

1.3 Outline of the paper

Section 2 defines the class of nonlinear systems to be investigated, states the control aim in detail and introduces a reference system with ideal disturbance rejection behavior. A novel design method for the event generator and control input generator is proposed in Section 3. Section 4 presents a disturbance estimator that is used in the event-based control scheme. Section 5 analyzes the performance of the closed-loop system and shows that the deviation of the behavior of the event-based control system to a continuous reference system is bounded. Section 6 illustrates the behavior of the event-based control method applied to a cooling process by simulation results.

2. PROBLEM STATEMENT

2.1 Plant

The plant is described by the affine state-space model

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_{\mathrm{x}}(\boldsymbol{x}(t)) + \boldsymbol{g}_{\mathrm{x}}(\boldsymbol{x}(t))u(t) + \boldsymbol{d}_{\mathrm{x}}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

$$y(t) = h(\boldsymbol{x}(t)), \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ denotes the state vector, $u \in \mathbb{R}$ is the input and $y \in \mathbb{R}$ is the output. The disturbance is represented by $\boldsymbol{d}_{\mathrm{x}} \in \mathcal{D}$ with $\mathcal{D}$ being a compact subset of $\mathbb{R}^n$ containing the origin. $\boldsymbol{f}_{\mathrm{x}}$ and $\boldsymbol{g}_{\mathrm{x}}$ are $\mathbb{R}^n$-valued mappings with $\boldsymbol{f}_{\mathrm{x}}$ satisfying the relation $\boldsymbol{f}_{\mathrm{x}}(\boldsymbol{0}) = \boldsymbol{0}$. The state $\boldsymbol{x}(t)$ is assumed to be measurable.

The plant (1), (2) is assumed to have the relative degree $r = n$ in the relevant part Ω_{z} of the state space defined later and is, thus, input-output linearizable in Ω_{z} (Isidori (1995)). Therefore, it can be described in the coordinates

$$\boldsymbol{z}(t) = (z_1(t), ..., z_n(t))^{\mathrm{T}} = \left(y(t), \dot{y}(t), ..., y^{(n-1)}(t)\right)^{\mathrm{T}}$$

after applying the transformation

$$\boldsymbol{z}(t) = \boldsymbol{\phi}(\boldsymbol{x}(t)), \tag{3}$$

where $\boldsymbol{\phi}$ denotes a mapping $\boldsymbol{\phi} : \mathbb{R}^n \to \mathbb{R}^n$. Applying the transformation (3) to Eq. (1) yields the representation of the plant in normal form

$$\dot{\boldsymbol{z}}(t) = \begin{pmatrix} z_2(t) \\ \vdots \\ z_n(t) \\ b(\boldsymbol{z}(t)) \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ a(\boldsymbol{z}(t)) \end{pmatrix} u(t) + \boldsymbol{d}(t), \tag{4}$$

$$\boldsymbol{z}(0) = \boldsymbol{z}_0 = \boldsymbol{\phi}(\boldsymbol{x}_0),$$

$$y(t) = z_1(t),$$

where $b(\boldsymbol{z}(t))$ and $a(\boldsymbol{z}(t))$ are nonlinear functions and $\boldsymbol{d}(t)$ is the transformed disturbance. Note that

$$a(\boldsymbol{z}(t)) \neq 0, \quad \forall \boldsymbol{z} \in \mathbb{R}^n, t \geq 0$$

holds, which means that the transformation (3) is globally valid. Subsequently, the compact form

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}(\boldsymbol{z}(t))u(t) + \boldsymbol{d}(t), \quad \boldsymbol{z}(0) = \boldsymbol{z}_0 \tag{5}$$

will be used instead of Eq. (4). Since the transformation (3) holds for all t, the state $\boldsymbol{z}(t)$ is also measurable.

2.2 Control aim

The main goal of the investigated event-based control scheme is the rejection of the disturbance $\boldsymbol{d}(t)$ in order to keep the state $\boldsymbol{z}$ of the plant (5) in a bounded surrounding of the setpoint $\overline{\boldsymbol{z}}$, which is assumed to be $\overline{\boldsymbol{z}} = \boldsymbol{0}$. The event-based control loop should be ultimately bounded, i. e.

$$\boldsymbol{z}(t) \in \Omega_{\mathrm{z}} \subset \mathbb{R}^n, \quad \forall t \geq 0 \tag{6}$$

with

$$\overline{\boldsymbol{z}} \in \Omega_{\mathrm{z}}$$

is required (Blanchini (1994)).

2.3 Reference system

The linearizable plants disturbance rejection can be accomplished by the linearizing state feedback

$$u(t) = \frac{1}{a(\boldsymbol{z}(t))}\left(-b(\boldsymbol{z}(t)) + v(t)\right) \tag{7}$$

with the new input $v(t)$, which is determined by the control law

$$v(t) = -\boldsymbol{k}^{\mathrm{T}}\boldsymbol{z}(t). \tag{8}$$

The static state-feedback gain $\boldsymbol{k}^{\mathrm{T}}$ is designed such that the closed-loop system is stable and requirements concerning the disturbance rejection are satisfied. Combining Eqs. (7) and (8) yields the nonlinear control law

$$u(t) = \frac{1}{a(\boldsymbol{z}(t))}\left(-b(\boldsymbol{z}(t)) - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{z}(t)\right), \tag{9}$$

which is applied to the plant (4). The resulting closed-loop system is described by the linear model

$$\dot{\boldsymbol{z}}(t) = \begin{pmatrix} z_2(t) \\ \vdots \\ z_n(t) \\ -\boldsymbol{k}^{\mathrm{T}}\boldsymbol{z}(t) \end{pmatrix} + \boldsymbol{d}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{d}(t), \quad \boldsymbol{z}(0) = \boldsymbol{z}_0 \tag{10}$$

with

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{pmatrix}, \tag{11}$$

where

$$k_i > 0, \quad \forall i = 1, ..., n$$

denotes the i-th element of the state-feedback gain $\boldsymbol{k}^{\mathrm{T}}$.

Equation (10) represents a linear reference system for the event-based control loop with desired disturbance rejection behavior. The main aim of the next section is to find an event-based feedback that has a similar disturbance behavior as this reference system.

The system (10) is input-to-state stable. Thus, there exist $\theta \in \mathcal{KL}$, $\gamma \in \mathcal{K}_\infty$, such that the solution to (10) is bounded by

$$||\boldsymbol{z}(t)|| \leq \theta(||\boldsymbol{z}_0||, t) + \gamma(||\boldsymbol{d}||_\infty), \qquad (12)$$

where $||.||$ denotes an arbitrary vector norm and

$$||\boldsymbol{d}||_\infty := \operatorname*{ess\,sup}_{t \geq 0} ||\boldsymbol{d}(t)|| \,.$$

In other words, the reference system (10) is ultimately bounded with

$$\Omega_{\mathrm{z,SF}} = \{\boldsymbol{z} | \, ||\boldsymbol{z}(t)|| \leq \theta(||\boldsymbol{z}_0||, t) + \gamma(||\boldsymbol{d}||_\infty)\} \,. \qquad (13)$$

3. NONLINEAR EVENT-BASED CONTROL LOOP

3.1 Control input generator

In dependence upon the information received at event time t_k, the control input generator determines the function $u(t)$ for the time interval $t \in [t_k, t_{k+1})$. At each event time t_k, a communication is invoked, where the event generator sends the time t_k, the state

$$\boldsymbol{z}(t_k) = \boldsymbol{\phi}(\boldsymbol{x}(t_k))$$

and a disturbance estimate $\hat{\boldsymbol{d}}_k$ to the control input generator. The state information is used to reinitialize the model of the plant

$$\begin{aligned} \dot{\boldsymbol{z}}_{\mathrm{s}}(t) &= \boldsymbol{f}(\boldsymbol{z}_{\mathrm{s}}(t)) + \boldsymbol{g}(\boldsymbol{z}_{\mathrm{s}}(t))u(t) + \hat{\boldsymbol{d}}_k \\ \boldsymbol{z}_{\mathrm{s}}(t_k^+) &= \boldsymbol{z}(t_k) \end{aligned} \qquad (14)$$

which is incorporated in the control input generator. Here, t_k^+ denotes the time instant after the state reset. The control input $u(t)$ for the time interval $t \in [t_k, t_{k+1})$ is then determined by means of the control law (9) with $\boldsymbol{z}(t) = \boldsymbol{z}_{\mathrm{s}}(t)$:

$$u(t) = \frac{1}{a(\boldsymbol{z}_{\mathbf{s}}(t))} \left(-b(\boldsymbol{z}_{\mathbf{s}}(t)) - \boldsymbol{k}^{\mathrm{T}} \boldsymbol{z}_{\mathrm{s}}(t)\right). \qquad (15)$$

The structure of the control input generator (14), (15) is shown in Fig. 2. The generator is a linear system described by

$$\dot{\boldsymbol{z}}_{\mathrm{s}}(t) = \boldsymbol{A}\boldsymbol{z}_{\mathrm{s}}(t) + \hat{\boldsymbol{d}}_k, \quad \boldsymbol{z}_{\mathrm{s}}(t_k^+) = \boldsymbol{z}(t_k),$$

with matrix $\boldsymbol{A}$ given by Eq. (11).

Note that if the relation

$$\hat{\boldsymbol{d}}_k = \boldsymbol{d}(t), \quad \forall t \geq t_k$$

holds, $\boldsymbol{z}(t) = \boldsymbol{z}_{\mathrm{s}}(t)$ is satisfied and the event-based control loop behaves exactly like the reference system (10). This fact shows that the determination of the control input $u(t)$ according to Eq. (15) is an appropriate choice.

3.2 Event generator

The event generator should indicate time instants at which an information feedback is necessary. Since the structure of the control input generator is changed compared to linear event-based control, the event generation condition that detects these time points needs to be renewed as follows. The event-based control loop with the plant (4) under the control input (15) is described by the state-space model

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{e}_n \mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) + \boldsymbol{d}(t), \quad \boldsymbol{z}(0) = \boldsymbol{z}_0 \quad (16)$$

with

$$\boldsymbol{e}_n = (0 \; \dots \; 0 \;\; 1)^{\mathrm{T}}$$

and

$$\begin{aligned} \mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) &= \beta(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) + \boldsymbol{k}^{\mathrm{T}} \boldsymbol{\alpha}(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) \\ \beta(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) &= b(\boldsymbol{z}(t)) - \frac{a(\boldsymbol{z}(t))}{a(\boldsymbol{z}_{\mathrm{s}}(t))} b(\boldsymbol{z}_{\mathrm{s}}(t)) \\ \boldsymbol{\alpha}(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) &= \boldsymbol{z}(t) - \frac{a(\boldsymbol{z}(t))}{a(\boldsymbol{z}_{\mathrm{s}}(t))} \boldsymbol{z}_{\mathrm{s}}(t). \end{aligned}$$

A comparison of Eq. (16) with the reference system (10) points out that the ideal disturbance rejection behavior is obtained for

$$\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) = 0. \qquad (17)$$

This condition can generally only be kept by infinitely fast sampling, ensuring the equality $\boldsymbol{z}_{\mathrm{s}}(t) = \boldsymbol{z}(t)$ and, hence,

$$\beta(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) = 0, \qquad \boldsymbol{\alpha}(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) = \boldsymbol{0}.$$

However, the aim of event-based control is to restrict the sampling. Hence, it is unavoidable that the function $\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t))$ takes nonzero values due to a deviation between $\boldsymbol{z}(t)$ and $\boldsymbol{z}_{\mathrm{s}}(t)$. The idea of event generation is to limit the growth of the function $\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t))$. An event is generated at time t_k if

$$|\mu(\boldsymbol{z}(t_k), \boldsymbol{z}_{\mathrm{s}}(t_k))| = \bar{e} \qquad (18)$$

is satisfied, with the event generation threshold $\bar{e} \in \mathbb{R}_+$. At time t_k, the current plant state $\boldsymbol{z}(t_k)$ and the disturbance estimate $\hat{\boldsymbol{d}}_k$ are sent to the control input generator. After state resetting at time t_k,

$$\mu(\boldsymbol{z}(t_k^+), \boldsymbol{z}_{\mathrm{s}}(t_k^+)) = 0$$

holds with $\boldsymbol{z}_{\mathrm{s}}(t_k^+) = \boldsymbol{z}(t_k)$. Hence, for all time t the following relation holds, as well:

$$|\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t))| \leq \bar{e}. \qquad (19)$$

Moreover, at time $t_0 = 0$, an initial event is triggered independently of the condition (18), in order to let the state $\boldsymbol{z}_{\mathrm{s}}(0)$ of the model (14) coincide with the plant state $\boldsymbol{z}(0)$.

Subsequently the function $\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t))$ will be referred to as *event function* and Eq. (18) denotes the event generation condition.

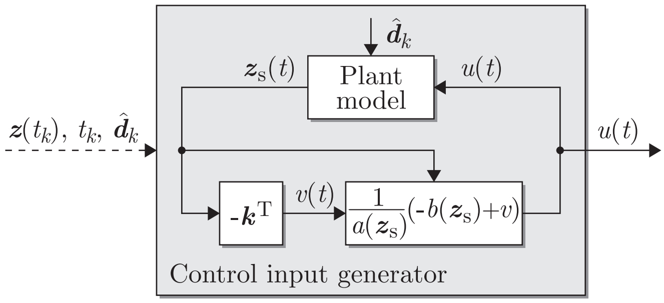

Fig. 2. Control input generator

3.3 Closed-loop system

The nonlinear event-based control system consists of the following components:

- the plant (5),
- the event generator, which triggers an event if the condition (18) is met, and
- the control input generator (14), (15).

If the k-th event is generated at time t_k, the information $\boldsymbol{z}(t_k), \hat{\boldsymbol{d}}_k$ is communicated from the event generator to the control input generator. The method how the event generator determines the disturbance estimate $\hat{\boldsymbol{d}}_k$ is described in Section 4. Between consecutive event times t_k and t_{k+1}, the event-based control works in an open-loop fashion.

4. DISTURBANCE ESTIMATION

The components of the event-based control loop, introduced in the preceding section work with an arbitrary disturbance estimation $\hat{\boldsymbol{d}}_k$, including the trivial one ($\hat{\boldsymbol{d}}_k = \boldsymbol{0}$). This section proposes a disturbance estimator, which is based on the assumption that the disturbance $\boldsymbol{d}(t)$ in (5) is slowly varying or piecewise constant in the time interval $[t_k, t_{k+1})$ and can, therefore, be approximated by the constant vector $\overline{\boldsymbol{d}}$:

$$\boldsymbol{d}(t) \approx \overline{\boldsymbol{d}}, \quad t \in [t_k, t_{k+1}).$$

The plant model (5) then simplifies to

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}(\boldsymbol{z}(t))u(t) + \overline{\boldsymbol{d}}.$$

The solution to this differential equation for $t \geq t_k$ is given by

$$\boldsymbol{z}(t) = \boldsymbol{z}(t_k) + \int_{t_k}^{t} \boldsymbol{f}(\boldsymbol{z}(\tau)) + \boldsymbol{g}(\boldsymbol{z}(\tau))u(\tau)\mathrm{d}\tau + \int_{t_k}^{t} \overline{\boldsymbol{d}}\,\mathrm{d}\tau$$

and for the next event time t_{k+1}

$$\begin{aligned}\boldsymbol{z}(t_{k+1}) = \boldsymbol{z}(t_k) \; &+ \int_{t_k}^{t_{k+1}} \boldsymbol{f}(\boldsymbol{z}(\tau)) + \boldsymbol{g}(\boldsymbol{z}(\tau))u(\tau)\mathrm{d}\tau \\ &+ (t_{k+1} - t_k)\,\overline{\boldsymbol{d}}\end{aligned}$$

holds. This equation is employed to determine the disturbance vector $\overline{\boldsymbol{d}}$ by

$$\begin{aligned}\overline{\boldsymbol{d}} \;=\; &\frac{1}{t_{k+1} - t_k}\Bigg[\boldsymbol{z}(t_{k+1}) \\ &- \left(\boldsymbol{z}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{f}(\boldsymbol{z}(\tau)) + \boldsymbol{g}(\boldsymbol{z}(\tau))u(\tau)\mathrm{d}\tau\right)\Bigg],\end{aligned} \tag{20}$$

which is used as the estimate $\hat{\boldsymbol{d}}_{k+1}$ for the next time interval $t \geq t_{k+1}$.

Note that

$$\boldsymbol{z}_{\mathrm{e}}(t_{k+1}) = \boldsymbol{z}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{f}(\boldsymbol{z}(\tau)) + \boldsymbol{g}(\boldsymbol{z}(\tau))u(\tau)\mathrm{d}\tau$$

is the solution to the differential equation

$$\dot{\boldsymbol{z}}_{\mathrm{e}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}(\boldsymbol{z}(t))u(t), \quad \boldsymbol{z}_{\mathrm{e}}(t_k) = \boldsymbol{z}(t_k) \tag{21}$$

at the event time t_{k+1}. Consequently, the disturbance estimator incorporates the system (21), with the measured plant state $\boldsymbol{z}(t)$ and the known control signal $u(t)$ as inputs. At the next event time t_{k+1}, the disturbance estimate $\hat{\boldsymbol{d}}_k$ is determined according to Eq. (20) by

$$\hat{\boldsymbol{d}}_{k+1} = \frac{1}{t_{k+1} - t_k}\left(\boldsymbol{z}(t_{k+1}) - \boldsymbol{z}_{\mathrm{e}}(t_{k+1})\right), \quad \hat{\boldsymbol{d}}_0 = \boldsymbol{0}$$

and is used for the time interval $t \geq t_{k+1}$. Since for this disturbance estimation method, the plant state $\boldsymbol{z}(t)$ has to be measured continuously and the control input $u(t)$ has to be known, the estimator needs to be implemented in the event generator. To this end, the generator also has to incorporate the model (14), (15).

5. ANALYSIS OF THE CLOSED-LOOP SYSTEM

This section considers the difference of the behavior of the event-based control loop represented by Eq. (16) and of the reference system (10) in dependence on the event threshold $\bar{e}$. For this purpose, the state of the continuous reference system will be denoted by $\boldsymbol{z}_{\mathrm{SF}}$:

$$\dot{\boldsymbol{z}}_{\mathrm{SF}}(t) = \boldsymbol{A}\boldsymbol{z}_{\mathrm{SF}}(t) + \boldsymbol{d}(t), \quad \boldsymbol{z}_{\mathrm{SF}}(0) = \boldsymbol{z}_0. \tag{22}$$

For the difference state

$$\boldsymbol{\delta}(t) = \boldsymbol{z}(t) - \boldsymbol{z}_{\mathrm{SF}}(t) \tag{23}$$

the equation

$$\dot{\boldsymbol{\delta}}(t) = \boldsymbol{A}\boldsymbol{\delta}(t) + \boldsymbol{e}_n\mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)), \quad \boldsymbol{\delta}(0) = \boldsymbol{0} \tag{24}$$

represents the dynamics of the difference between the behavior of the event-based control system (16) and the reference system (22).

Theorem 1. The difference $\boldsymbol{\delta}(t)$ between the event-based control loop (16) and the continuous reference system (22) is bounded from above by

$$||\boldsymbol{\delta}(t)|| \leq \delta_{\max}$$

with

$$\delta_{\max} = \bar{e} \cdot \int_0^{\infty} \left|\left|\mathrm{e}^{\boldsymbol{A}\tau}\boldsymbol{e}_n\right|\right| \mathrm{d}\tau. \tag{25}$$

Proof. Equation (24) yields the difference

$$\boldsymbol{\delta}(t) = \int_0^t \mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{e}_n\mu(\boldsymbol{z}(\tau), \boldsymbol{z}_{\mathrm{s}}(\tau))\mathrm{d}\tau,$$

for which an upper bound exists, since the matrix $\boldsymbol{A}$ is stable and (19) holds. Hence, a bound on the difference $\boldsymbol{\delta}(t)$ is given by

$$\begin{aligned}||\boldsymbol{\delta}(t)|| &\leq \int_0^t \left|\left|\mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{e}_n\right|\right| |\mu(\boldsymbol{z}(\tau), \boldsymbol{z}_{\mathrm{s}}(\tau))|\,\mathrm{d}\tau \\ &\leq \bar{e} \cdot \int_0^t \left|\left|\mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{e}_n\right|\right| \mathrm{d}\tau \\ &\leq \bar{e} \cdot \int_0^{\infty} \left|\left|\mathrm{e}^{\boldsymbol{A}\tau}\boldsymbol{e}_n\right|\right| \mathrm{d}\tau = \delta_{\max}.\end{aligned} \tag{26}$$

$\square$

Remark 1. The bound of the difference between the behavior of the event-based control loop (16) and the reference system (22) can be made arbitrarily small by scaling down the event threshold $\bar{e}$. A smaller event threshold however increases the communication frequency.

Remark 2. The theorem shows that the state of the nonlinear event-based control system always remains in a surrounding of the reference system (10):

$$\boldsymbol{z}(t) \in \Omega_\delta(\boldsymbol{z}_{\mathrm{SF}}(t)) = \{\boldsymbol{z}(t)|\, ||\boldsymbol{z}(t) - \boldsymbol{z}_{\mathrm{SF}}(t)|| \leq \delta_{\max}\}.$$

Since the reference system (10) is ultimately bounded according to (12) and the deviation between the reference system (10) and the event-based control loop (16) is bounded from above by (25), the event-based control loop is ultimately bounded as well. The surrounding Ω_z, in which the state $\boldsymbol{z}(t)$ remains for all $t \geq 0$, can be deduced from Eq. (13) and is given by

$$\Omega_z = \{\boldsymbol{z} | \, ||\boldsymbol{z}|| \leq \theta(||\boldsymbol{z}_0||, t) + \gamma(||\boldsymbol{d}||_\infty) + \delta_{\max}\}. \quad (27)$$

Accordingly, the event-based controlled system (16) satisfies the control aim (6) with the bounded set Ω_z defined by (27).

Remark 3. The theorem can be extended to obtain a bound for the difference between the output $y(t) = z_1(t)$ of the event-based control loop and the output $y_{SF}(t) = z_{SF,1}(t)$ of the reference system

$$|y_\Delta(t)| = |y(t) - y_{SF}(t)| \leq \bar{e} \cdot \int_0^\infty \left| \boldsymbol{e}_1^T e^{\boldsymbol{A}\tau} \boldsymbol{e}_n \right| d\tau$$

with $\boldsymbol{e}_1 = (1 \; 0 \; \dots \; 0)^T$. This result gives an indication on how to choose the event threshold $\bar{e}$.

6. EXAMPLE

6.1 Cooling process model

The proposed event-based control method is now illustrated by its application to the cooling process depicted in Fig. 3. The process consists of a cylindrical tank, surrounded by a cooling jacket. The tank is fed by a constant inflow $q_{t,in}$, which has the constant temperature $\vartheta_{in} = 24\,°C$. For the outflow $q_{t,out} = q_{t,in}$ is assumed, such that the filling level of the tank is constant. The cooling jacket is used to lower the temperature of the fluid in the tank. The inflow $q_{c,in}$ of the coolant with constant temperature $\vartheta_{CU} = 5\,°C$ is used as the input $u(t)$, in order to keep the temperature ϑ_t of tank at the desired value of $\vartheta_t = 20\,°C$. The temperature of the coolant in the cooling jacket is denoted by ϑ_c. Desired disturbance characteristics can be realized by means of the heating rods, which directly influence the temperature ϑ_t of the liquid in the tank.

The process has the measurable state variables

$$(x_1 \; x_2)^T = (\vartheta_t \; \vartheta_c)^T$$

and is described by the model

$$\dot{\boldsymbol{x}}(t) = 10^{-3} \begin{pmatrix} 0.6 \cdot x_1(t) - 1.31 \cdot x_2(t) + 2.14 \\ -4.6 \cdot x_1(t) + 4.6 \cdot x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -3.75 - 0.25 \cdot x_2(t) \end{pmatrix} u(t) + 10^{-4} \begin{pmatrix} 3.57 \cdot p \\ 0 \end{pmatrix}$$

$$y(t) = x_1(t), \quad (28)$$

where $p \in \{0, 1, 2, 3, 4\}$ denotes the number of used heating rods. With the local coordinate transformation

$$\boldsymbol{z}(t) = 10^{-3} \begin{pmatrix} 1000 \cdot x_1(t) \\ 0.6 \cdot x_1(t) - 1.31 \cdot x_2(t) + 2.14 \end{pmatrix} \quad (29)$$

the system (28) is represented in normal form by

$$\dot{\boldsymbol{z}}(t) = \begin{pmatrix} z_2(t) \\ b(\boldsymbol{z}(t)) \end{pmatrix} + \begin{pmatrix} 0 \\ a(\boldsymbol{z}(t)) \end{pmatrix} \cdot u(t) + 10^{-4} \begin{pmatrix} 3.57 \cdot p \\ 0 \end{pmatrix},$$

with

$$b(\boldsymbol{z}(t)) = 10^{-4} \, (0.033 \cdot z_1(t) - 52 \cdot z_2(t) + 0.098) \quad (30)$$

and

$$a(\boldsymbol{z}(t)) = 10^{-4} \, (1.5 \cdot z_1(t) - 2500 \cdot z_2(t) + 54.64). \quad (31)$$

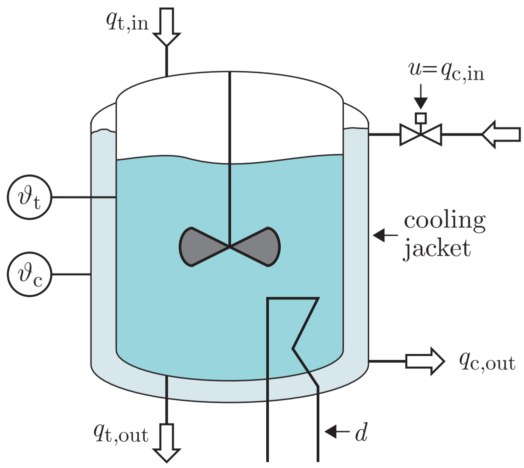

Fig. 3. Cooling process

Note that the transformation (29) causes a displacement of the origin, such that the inverse transformation of (29) yields

$$\boldsymbol{\phi}^{-1}(\boldsymbol{0}) = (0 \quad 1.63)^T.$$

This displacement only affects the temperature $x_2 = \vartheta_c$ of the coolant, in terms of an offset of the steady-state, even in the case of no disturbance. To compensate for this, a more sophisticated control method like asymptotic output tracking (Khalil (2002)) instead of a state feedback would have to be used. However, in this example the control aim is to keep the temperature x_1 of the liquid in the tank at a constant level, which is not endangered by the offset of the temperature x_2 of the coolant. Therefore, a state-feedback controller is applied with the state-feedback gain

$$\boldsymbol{k}^T = (0.0026 \quad 0.11), \quad (32)$$

which is designed such that the continuously controlled system has a satisfactory disturbance rejection behavior.

The event generator uses the event threshold

$$\bar{e} = 1 \cdot 10^{-4}$$

to determine the event time instants. With this parameter and the state-feedback gain (32), the maximal deviation between the reference system and the event-based control loop according to (25) results to

$$\delta_{\max} = 0.043. \quad (33)$$

6.2 Simulation results

Constant disturbance. First, the behavior of the event-based control loop is investigated in case of a constant disturbance

$$d_1(t) = 10^{-4}(3.57 \cdot p)$$

with $p = 3$. (left-hand side of Fig. 4). An event is generated at time t_1, when the trigger condition (18) is satisfied, indicated by a stem in the bottom subplot. At this time, the disturbance $\boldsymbol{d}(t)$ is estimated correctly and the event generator sends the estimation $\hat{\boldsymbol{d}}_1$ and the plant state $\boldsymbol{z}(t_1)$ to the control input generator. Since from this time on, the disturbance $d_1(t)$ does not change, the model state $\boldsymbol{z}_s(t)$ and the plant state $\boldsymbol{z}(t)$ are identical and no further event is generated. At steady-state the behavior of the event-based control loop coincides with the one of the reference system (cf. second and third subplot from the top).

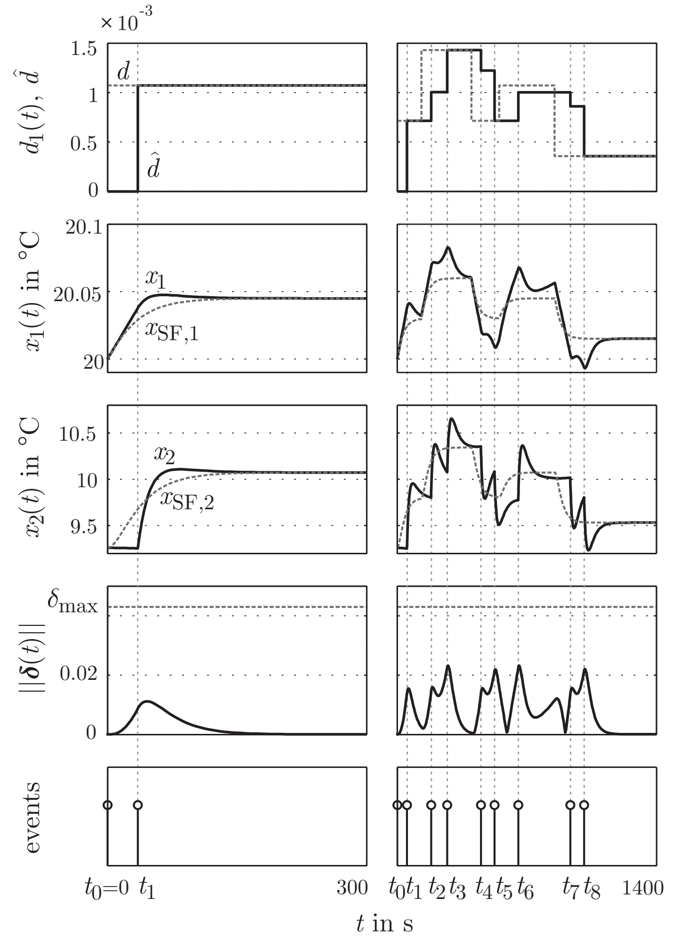

Fig. 4. Behavior of the event-based control loop in case of constant disturbance (left) and piecewise constant disturbance (right).

In this case of constant disturbance, only one feedback communication, except the initial one, is induced. The transferred information is sufficient in order to keep the deviation between the plant state $\boldsymbol{z}(t)$ and the state of the reference system $\boldsymbol{z}_{\mathrm{SF}}(t)$ below the bound (33), as shown by the second subplot from the bottom.

Time-varying disturbance. The second investigation (right-hand side of Fig. 4) concerns the behavior of the event-based control loop subject to a time-varying disturbance $d_1(t)$. Over the period of 1400 seconds, eight events, except the initial one, are generated.

Similarly to the first investigation, an event is triggered at time $t = t_1$ and the disturbance is correctly estimated. Since the disturbance varies, a new event at time $t = t_2$ is triggered. At this second event time however, the disturbance estimate $\hat{\boldsymbol{d}}_2$ is a weighted average of the disturbance $d_1(t)$ in the preceding interval $[t_1, t_2)$. The estimation value $\hat{\boldsymbol{d}}_1$ and the real disturbance $\boldsymbol{d}(t)$ do not coincide, until a new event at time $t = t_3$ is generated and the disturbance $\boldsymbol{d}(t)$ is estimated correctly again. After the event at time $t = t_8$, the disturbance $\boldsymbol{d}(t)$ remains constant and the estimation value $\hat{\boldsymbol{d}}_8$ coincides with its magnitude, hence no further events are generated. At steady-state, the states of the event-based control loop and of the reference system coincide.

In this case, the control aim is achieved by initiating a communication at eight time instants. Compared with the continuous reference system, a satisfactory disturbance rejection behavior of the event-based control loop was preserved, since the deviation of the dynamics of both systems remains in the derived range (33) (cf. second subplot from the bottom).

7. CONCLUSION

The paper proposed a new event-based control scheme for nonlinear systems. The difference between the behavior of the event-based control loop and a continuous state-feedback control loop with ideal disturbance rejection was shown to be bounded. It can be made arbitrarily small by appropriately downsizing the event threshold parameter of the event generator.

Simulation results of the application of the event-based control method to a cooling process indicated that the feedback communication effort is considerably reduced comparing to a continuous state feedback, while preserving a satisfactory disturbance rejection behavior.

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